# PASSIVE LOCATION METHOD BASED ON PHASE DIFFEREDES MEASUREMENT 



Tao Yu
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# Passive Location Method Based on Phase Difference Measurement 

Authored by<br>\section*{Tao Yu}<br>China Institute of Aeronautical Radio Electronics<br>China

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## PREFACE

The author began to work on radiolocation technology around 2005, when author joined the China Aeronautical Radio Electronics Research Institute. Working on radiolocation technology was only a fortuitous opportunity. In fact, the author had not done any research related to location technology before this time. But the author didn't do some specious writing, but hard work and research. And after five years, in the choice between continuing the research of positioning technology or changing careers again, I self-evaluated that I might be more suitable for positioning technology, and made a fruitful choice along with my own will.

Phase difference positioning is a branch of radio positioning technology. It may be difficult to solve the problems in the phase difference positioning equation with periodic variation by using conventional methods. The author's research on phase difference passive location method started from the virtual baseline direction finding method. At that time, the author completed the research work on the linear solution of the double base path differential positioning equation and is trying to further study the phase differential positioning equation by using the relationship between the radial distance and the phase. At this time, the author just studied the virtual baseline direction finding method. The author keenly noticed the phase jump problem in the process of constructing the virtual short baseline, analyzed in detail the problem that the phase ambiguity value of the virtual direction finding method is not zero, and gave a method to correct it by judging the sine value of the angle of arrival. This research result strongly inspired the author. The author's main innovative contribution on unambiguous phase difference positioning lies in:

1. The concept of the differential function of the path difference per unit wavelength length is put forward.
2. The jump law of the differential function is discovered and the method to correct the jump is found.
3. The results of the research on the intrinsic properties of symmetric arrays enhance the extended-application of the unambiguous phase difference positioning technique.

This monograph not only includes some of the author's existing research results, but also gives the author's many new opinions. I hope that this monograph will contribute to the development of phase difference passive localization technology.

## CONSENT FOR PUBLICATION

None.

## CONFLICT OF INTEREST

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## CHAPTER 1

## The General Solution of the Phase Difference Positioning Equation


#### Abstract

For passive positioning technology, not only the signal transmitting time of the target is not known at all, but also the initial phase of the signal is not known. Therefore, the phase comparison method cannot be used. But if we just use geometry, starting from the basic definition of phase shift detection of distance, it is explained in mathematical form that, because phase shift measurement has the fuzziness of period, the number of wavelengths contained in the observed quantity itself is an unknown variable to be determined, so the positioning equation based on phase shift measurement is unsolvable if we only use existing analytical methods. If the unknown quantity representing the period ambiguity is regarded as an undetermined quantity, the solution of the phase difference localization equation can still be obtained formally. As a mathematical basis, this chapter gives a linear solution method of passive positioning equation based on path difference measurement in two dimensional plane. In the passive positioning problem based on path difference measurement in two-dimensional plane, it is generally necessary to use at least 3 or more measuring stations to collect data to get the path difference between the radiation source and each measuring station. The existing method is to make use of these distance differences to form a set of nonlinear hyperbolic equations about the position of the radiation source, and the coordinate position of the radiation source can be obtained by solving the hyperbolic equations. The author's existing research results show that for the plane multi-station positioning problem, the linear equations can be obtained if the auxiliary equations are constructed by using the existing plane geometric relations on the basis of the path difference measurement.


Keywords: Linear equation, One-dimensional double-base linear array, Path difference, Phase ambiguity, Phase comparison method, Phase difference localization equation, Phase shift.

### 1.1. INTRODUCTION

In this chapter, the general solution of the phase difference positioning equation involving one-dimensional double-base array is given.

Starting from the basic definition of phase shift detection of distance, it is explained in a mathematical form that, because phase shift measurement has the fuzziness of period, the number of wavelengths contained in the observed quantity itself is an

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unknown variable to be determined, so the positioning equation based on phase shift measurement is unsolvable if we only use existing analytical methods.

If the unknown quantity representing the period ambiguity is regarded as an undetermined quantity, the solution of the phase difference localization equation can still be obtained formally. The author calls this kind of solution which still contains an unknown quantity the general solution of the phase difference positioning equation by using the existing mathematical terminology.

From the perspective of pure mathematics, this chapter actually studies the linear solution of the passive positioning equation based on the measurement of the path difference on the two-dimensional plane, regardless of which method is used to measure the path difference. In the passive positioning problem based on path difference measurement in two-dimensional plane, it is generally necessary to use at least 3 or more measuring stations to collect data to get the path difference between the radiation source and each measuring station. The existing method is to make use of these distance differences to form a set of nonlinear hyperbolic equations about the position of the radiation source, and the coordinate position of the radiation source can be obtained by solving the hyperbolic equations [1-9].

The traditional method of solving nonlinear equations is based on iterative operation and linearization, and the accuracy of the solution method strongly depends on whether the estimation of the initial position is accurate. When the initial estimation is poor, the convergence solution may not be obtained, and the calculation amount of this estimation method is also very large. In addition, in the process of location solution, because hyperbola (surface) intersections sometimes appear as location ambiguity, seeking a method to remove the location ambiguity is also a problem that needs to be solved [10-14].

As the analytical basis of this monograph, this chapter mainly introduces the author's existing research results [23]. The author's existing research results show that for the plane multi-station positioning problem, the linear equations can be obtained if the auxiliary equations are constructed by using the existing plane geometric relations on the basis of the path difference measurement.

### 1.2. THE UNCERTAINTY OF PHASE SHIFT RANGING

In the case that the time of receiving and transmitting signals is known or the time difference between receiving and transmitting signals can be measured, the distance
of the measured target can be obtained by using the phase difference between receiving and transmitting signals directly through signal analysis [15-22].

But for passive positioning technology, not only the signal transmitting time of the target is not known at all, but also the initial phase of the signal is not known. Therefore, the phase comparison method cannot be used.

Based on basic physical concepts, the ranging formula for direct use of phase shift measurement is

$$
\begin{equation*}
r(t)=\lambda\left(n+\frac{\phi(t)}{2 \pi}\right) \tag{1.2-1}
\end{equation*}
$$

Where, $r(t)$ is the radial distance; $\lambda$ the wavelength; $n$ the number of wavelength; $\phi(t)$ the phase shift, $t$ time.

Because the number of wavelength is unknown, the ranging formula (1.2.1) can not be directly applied to engineering design. Moreover, the number of wavelengths is not a relatively fixed, invariant quantity, but a quantity that changes with distance. If the geometric conditions are used to establish the system of a definite solution.
$r_{k}(t)=\lambda\left(n_{k}+\frac{\phi_{k}(t)}{2 \pi}\right) \quad(k=0,1, \ldots, n)$
This means that the number of unknown undetermined variables, $n_{k}$, will increase simultaneously at the same time as adding a definite equation. Therefore, unless the definite equation is added by means of another measurement system, such as in combination with the time difference location method, the ranging equation based on the direct phase shift measurement directly from the mathematical equation is actually unsolvable.

### 1.3. PHASE DIFFERENCE DETECTION OF PATH DIFFERENCE

According to the relation (1.2-1) between phase-shift and distance, corresponding to the single baseline array as shown in Fig. (1.3-1), the path difference $\Delta r$ between the radial distances of the two array elements $S_{1}$ and $S_{2}$ can be determined by phase difference measurement, and the phase difference positioning equation,

## CHAPTER 2

## Internal Characteristics of Double-base Array


#### Abstract

One-dimensional double-base array is the most basic model of multistation passive positioning system. The study and analysis of this basic model can not only help to improve the design performance of the positioning system, but also help to improve the measurement accuracy of the positioning system. But so far, most of the researches on passive positioning system only arrange the mathematical equations from the perspective of solving the unknown quantity, without in-depth research on the internal correlation between each parameter. In fact, the one-dimensional linear array has many interesting inherent characteristics. Three intrinsic characteristics of one-dimensional double-base arrays are studied and given. Firstly, four methods of proving the midpoint direction finding solution on a single basis are described. Then, the arithmetic characteristics between adjacent paths are given. On this basis, some physical properties are briefly described. And then we can reveal the internal correlation between the difference of the path difference and other motion parameters as well as physical parameters. Finally, the median relationship among the three arrival angles and among the three radial distances is analyzed.


Keywords: Arithmetic characteristic, Arrival angle, Radial distance, Median relation, One-dimensional double-base array, Physical Characteristics, Radial distance, Single base midpoint direction finding.

### 2.1. INTRODUCTION

One dimensional dual base array is the most basic model of multi-site passive location system. On the basis of linear solutions of one-dimensional double-base symmetric arrays, the authors obtain three significant results:

First, on the basis of a simplified analysis of the one-dimensional double-base direction-finding solution, a single base midpoint direction finding formula which is suitable for both long and short baselines is obtained. Thus, a more complete and accurate mathematical description is given for the analysis and research of the problems related to the long baselines.

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Secondly, on the basis of the variable substitution of the one-dimensional double basis direction finding solution, we obtain the properties of arithmetic series between adjacent path difference, and then we can reveal the internal correlation between the difference of the path difference and other motion parameters as well as physical parameters.

Third, for the one-dimensional double-base symmetric array, there exists a median relation in both the angle of arrival and the radial distance.

### 2.2. SINGLE BASE MIDPOINT DIRECTION FINDING FORMULA

### 2.2.1. Overview

By approximately simplifying the one-dimensional two-base path difference direction finding solution in literature [1], a single base midpoint direction finding formula is given, whose form is very simple, and which is suitable for a long baseline. But in the derivation, the justification for the approximation applied does not seem very strong. This chapter first describes the existing formula simplification methods. Then, starting from the physical definition of path difference directly, the radial distance and arrival angle of the midpoint of a single baseline is used as the basic parameters of the path difference equation, and the square root term of the path difference equation is simplified by using the differential approximation rule, so the radial distance can be exactly cancelled. Thus, the midpoint direction finding solution of a single baseline based on path difference measurement can be obtained directly. Then, the third method, based on the basic edge-angle relation in the location triangle, proves the single base midpoint direction finding solution again by using the median relation among the three arrival angles of a one-dimensional equidistant double basis line array.

Finally, a single basis direction finding solution related to intersection angle is obtained by means of the geometric projection method, which shows that using intersection angle between two stations can effectively improve the accuracy of a single basis midpoint direction-finding solution.

### 2.2.2. Formula Simplification Method

According to the analysis of the first chapter, one dimensional symmetric double basis direction finding solution is:

$$
\begin{equation*}
\sin \theta_{2}=\frac{\left(d^{2}-\Delta r_{12}^{2}\right) \Delta r_{23}+\left(d^{2}-\Delta r_{23}^{2}\right) \Delta r_{12}}{d\left(2 d^{2}-\Delta r_{12}^{2}-\Delta r_{23}^{2}\right)} \tag{1.5-13}
\end{equation*}
$$

If the higher order term of the path difference contained in the strict directionfinding solution is to simplify the approximate equality: $\Delta r_{12}^{2} \approx \Delta r_{23}^{2}$, then,

$$
\sin \theta_{2}=\frac{\left(d^{2}-\Delta r_{12}^{2}\right)\left(\Delta r_{12}+\Delta r_{23}\right)}{2 d\left(d^{2}-\Delta r_{12}^{2}\right)}=\frac{\Delta r_{12}+\Delta r_{23}}{2 d}
$$

Due to:

$$
\Delta r_{13}=r_{1}-r_{3}=\left(r_{1}-r_{2}\right)+\left(r_{2}-r_{3}\right)=\Delta r_{12}+\Delta r_{23}
$$

So there is a single baseline direction finding solution:
$\sin \theta_{2}=\frac{\Delta r_{13}}{2 d}$

### 2.2.3. Path Difference Approximation

The double-base array as shown in Fig. (1.5-1) was used for the deduction. According to the definition of path difference, there are

$$
\Delta r=r_{1}-r_{3}
$$

Among them

$$
\begin{aligned}
& r_{1}=\sqrt{r_{2}^{2}+d^{2}+2 r_{2} d \sin \theta_{2}} \\
& r_{3}=\sqrt{r_{2}^{2}+d^{2}-2 r_{2} d \sin \theta_{2}}
\end{aligned}
$$

## Path Difference and Its Difference Function


#### Abstract

This chapter is the most important one in this monograph. In this chapter, a detection method for phase change rate without phase ambiguity is presented. The results of this study lay a foundation for the realization of unambiguous phase difference localization. First, based on the functional relationship between the distance change rate and the phase shift change rate, the phase difference measurement method of the phase shift change rate is given through the difference approximation of the distance change rate. Then, the expression of phase difference change rate based on the multi-channel phase difference measurement is obtained by using the phase difference measurement of phase shift change rate and differential processing by phase differential rate of change. On this basis, by stripping the time difference term corresponding to the baseline length from the change rate of phase difference, a function representing the different characteristics of the number of wavelengths and the phase difference per unit wavelength length is extracted. The subsequent simulation results show that the variation of the difference function of path difference per unit wavelength length is very regular. The corresponding correction number can be determined directly by distinguishing the range of difference of phase difference, and the range is obtained by the actual measurement. A function expression can be obtained independent of the difference term of the integer of wavelengths as well as equivalent to the difference function of path difference. Finally, the problem of nonfuzzy phase difference measurement for path difference is briefly described. These research results undoubtedly provide a powerful technical support for the practical design of engineering related to phase measurement.


Keywords: Doppler shift, Passive localization, Path difference, Phase ambiguity, Phase difference rate, Phase interference.

### 3.1. INTRODUCTION

One of the important contents of this chapter is that based on the relation between the difference function of path difference and the change rate of phase difference discussed in the previous chapter, the non-fuzzy phase difference measurement method of the change rate of phase difference is studied and given here. Another

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important content of this chapter is to give the non-fuzzy measurement method of path difference by comprehensively utilizing the difference term of path difference per unit wavelength length and the single baseline midpoint direction finding solution [1].

This chapter is more about the detection of phase difference change rate without ambiguity. First, based on the functional relationship between the distance change rate and the phase shift change rate, the phase difference measurement method of the phase shift change rate is given through the difference approximation of the distance change rate. Then, the expression of phase difference change rate based on the multi-channel phase difference measurement is obtained by using the phase difference measurement of phase shift change rate and differential processing by phase differential rate of change. On this basis, by stripping the time difference term corresponding to the baseline length from the change rate of phase difference, a function representing the difference characteristics of the number of wavelengths and the phase difference per unit wavelength length is extracted. The subsequent simulation results show that the variation of the difference function of path difference per unit wavelength length is very regular. The corresponding correction number can be determined directly by distinguishing the range of difference of phase difference, that the range is obtained by the actual measurement. A function expression can be obtained independent of the difference term of the integer of wavelengths as well as equivalent to the difference function of path difference.

### 3.2. MULTI-CHANNEL PHASE DIFFERENCE DETECTION OF PHASE DIFFERENCE CHANGE RATE

### 3.2.1. Overview

The rate of change of phase difference can be used for passive positioning [2-6]. At present, there are two main methods to obtain the phase difference rate [7]: One is to extract the phase difference rate by measuring phase difference sequences and using algorithms such as difference, Kalman filtering and linear fitting [8]. The other is to calculate the phase difference rate by indirectly measuring the output frequency difference between two comparison channels based on phase discrimination [9, 10]. The previous method must be measured in a row to get enough phase difference sequences, and it needs to increase the measurement accuracy by extending the observation time. It's possible that the obtained phase difference is not linear. In the case of the short baseline application, the equivalent
measurement time is very short because the frequency difference interval that can be measured is too small so that the posterior method can make the measurement error become very large. Therefrom, the prediction accuracy of the phase difference rate is hard to improve.

### 3.2.2. Phase-Shift Change Rate by Detecting Phase Difference

### 3.2.2.1. Difference Approximation

By differentiating with respect to time on both sides of the phase-distance expressions (1.2-1), the relationship between the phase-shift change rate and the distance change rate can be obtained:

$$
\begin{equation*}
\frac{\partial r(t)}{\partial t}=\frac{\lambda}{2 \pi} \frac{\partial \phi(t)}{\partial t} \tag{3.2-1}
\end{equation*}
$$

Making a difference approximation for distance change rate:

$$
\begin{equation*}
\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r(t)}{\Delta t}=\frac{\lambda}{\Delta t}\left(\Delta n+\frac{\Delta \phi}{2 \pi}\right) \tag{3.2-2}
\end{equation*}
$$

The change rate of phase shift based on phase difference measurement can be obtained:

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}=\frac{2 \pi}{\Delta t}\left(\Delta n+\frac{\Delta \phi}{2 \pi}\right) \tag{3.2-3}
\end{equation*}
$$

### 3.2.2.2. Direction Finding Approximation

The change rate of radial distance is the radial velocity of the target:

$$
\begin{equation*}
\frac{\partial r}{\partial t}=v_{r}=v \sin \theta \tag{3.2-4}
\end{equation*}
$$

Where: $v_{r}$ is the radial speed.

Using the relationship (3.2-1) between the phase-shift change rate and the distance change rate, we can obtain:

## CHAPTER 4

# Unambiguous Phase Difference Measurement of Positioning Parameters 


#### Abstract

Through a simple angle substitution, we can obtain the corresponding relationship between frequency shift and phase difference. Based on this relationship, the Doppler shift can be obtained by simple phase difference measurement. However, the reason why the function relation between phase shift and frequency shift can produce the expansion efficiency is also due to the relationship, that is obtained by the first order change of distance, between the rate of change of phase shift and the Doppler frequency shift. It's just a simple mathematical generalization of what's already known. Further, based on the two basic modes of angle permutation and firstorder change, various relations between phase shift and frequency shift, as well as their first-order change, can be determined. This mathematical description extends to physical applications, enabling many motion parameters and observations that would otherwise be difficult to detect to be obtained by simple phase shift/difference detection.


Keywords: Angle change rate, Doppler change rate, Doppler frequency shift, Passive positioning, Path difference, Phase difference measurement, Phase difference rate, Radial acceleration.

### 4.1. UNAMBIGUOUS PHASE DIFFERENCE MEASUREMENT OF DOPPLER PARAMETERS

### 4.1.1. Introduction

As far as passive positioning technology is concerned, in order to quickly determine the position of maneuvering target, it is necessary not only to obtain the kinematic positioning parameters but also to use the signal waveform parameters. For example, the most common use is to use the Doppler shift function in the signal space. In fact, several kinematic positioning parameters have to be obtained by measuring the signal parameters.

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How to use each parameter of the signal waveform to realize the target location is the focus of passive location research. Especially for the localization of a moving target or a moving observation station, it is a difficult problem in the current research how to realize the target localization by using the relationship between the highly nonlinear Doppler frequency and the moving state of the target.

In this section, based on the functional relationship between phase shift and frequency shift, a method for detecting Doppler frequency difference and rate of change on a moving platform is presented based on unambiguous phase difference measurement technology.

### 4.1.2. Fuzzy - free Solution of Doppler Shift

### 4.1.2.1. Definition

As shown in Fig. (4.1.2-1), a Doppler receiver $R$ is installed on the moving platform to detect stationary or slow-moving target $T$ on the ground. The Doppler frequency shift received by the receiving array is:
$\lambda f_{d}=v \cos \beta$
Where: $f_{d}$ is Doppler frequency shift; $v$ the flying speed of the moving platform; $\beta$ the front angle.


Fig. (4.1.2-1). Detection of Doppler shift.

### 4.1.2.2. Function Relation between Frequency Shift and Phase Shift Change Rate

### 4.1.2.2.1. The Method of Deduction Based on Distance Change

According to the relationship between the change rate of phase shift and the change rate of distance:

$$
\begin{equation*}
\frac{\partial r(t)}{\partial t}=\frac{\lambda}{2 \pi} \frac{\partial \phi(t)}{\partial t} \tag{3.2-1}
\end{equation*}
$$

According to the relationship between the rate of change of radial distance and radial velocity:

$$
\begin{equation*}
\frac{\partial r}{\partial t}=v_{r}=v \sin \theta \tag{3.2-4}
\end{equation*}
$$

And the relationship between radial velocity and Doppler frequency shift (4.1.2-1)
$\lambda f_{d}=v \cos \beta$
It can be concluded that:

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}=2 \pi f_{d} \tag{4.1.2-2}
\end{equation*}
$$

Thus, the relationship between the change rate of radial distance and Doppler frequency shift can be obtained using this equation:

$$
\begin{equation*}
\frac{\partial r(t)}{\partial t}=\lambda f_{d} \tag{4.1.2-3}
\end{equation*}
$$

### 4.1.2.2.2. Analysis Results Based on Signal Waveform

As a contrast, this section presents existing representations based on signal waveforms.

Assume: there is a real band communication number $s(t)$

## CHAPTER 5

# Unambiguous Phase Difference Direction Finding Based on Short Baseline Array 


#### Abstract

In this chapter, the fuzzy-free phase difference direction finding method based on a short baseline array is introduced and three different methods are presented. Firstly, the virtual short baseline direction finding method based on onedimensional double-base asymmetrical array is studied deeply, which is constructed by subtraction of the ratio of different sides between two adjacent baselines. The author's findings show that, although the difference between the lengths of two adjacent baselines is less than half a wavelength, the difference of the integers of wavelengths will not be zero in the direction of partial arrival angles but will jump. In this regard, the correction can be realized by the determination of the sine value of the arrival angle by adopting a method like the fuzzy-free detection analysis of the phase difference rate. The second approach is the orthogonal phase difference direction finding method based on equivalent simulation. It is found in the simulation that the curve shape of the differential function of path difference per unit wavelength obtained after phase jump correction is very similar to that of the cosine function. If the maximum value of the function is used for normalization processing and simple square root processing, then the function obtained is basically equivalent to the cosine function. At this time, it can be proved in principle that the results given are equivalent to the Doppler direction finding technique. Then, using the orthogonal array, the maximum value of the function which cannot be known in the one-dimensional array is eliminated by means of the orthogonal ratio, so the real-time direction finding based on phase difference measurement without phase ambiguity is realized. The third approach is the airborne direction finding method based on Doppler-phase measurement. The study shows that the airborne single-baseline interferometer can achieve high precision direction finding without phase ambiguity after integrating Doppler measurement information. The main method is to directly obtain the wavelength integer solution of the radial distance by comprehensively utilizing the velocity vector equation, Doppler frequency shift and its rate of change. Thus, the integral value of the wavelength contained in the path difference between two adjacent array elements can be given. By means of the phase difference measurement, the value less than the wavelength integer in the path difference can be determined. This chapter also explores the effect of phase difference measurement errors on the difference of wavelength integers. The expression of the wavelength number difference based on the phase difference measurement can also be approximated by the unambiguous phase difference direction finding method based on the virtual short baseline. The root mean square measurement error of the wavelength number


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> difference is derived. Through analysis, it is revealed that the wavelength number difference has little effect on the accuracy of a single baseline phase difference direction finding.

Keywords: Virtual baseline, Phase interferometer, Phase jump, Phase ambiguous, Orthogonal array, Doppler DF, Direction finding, Doppler frequency, Airborne single station location, DF error.

### 5.1. INTRODUCTION

Phase interference measurement is a direction-finding method with high measurement accuracy and has been widely used in passive detection systems [18]. However, the single-baseline phase interferometer has a contradiction between the direction-finding accuracy and the maximum unambiguous angle. To solve this contradiction, a multi-baseline system is usually required, including the method combining long and short baselines and the multi-baseline unambiguous algorithm.

On the one hand, in practice, the method of combining long, and short baselines has two limitations. In fact, for high-frequency signals, when the wavelength is very short, the corresponding short baseline length will also become very small, which not only makes the antenna array element must be very small but also puts forward high requirements for the antenna layout and installation. As a result, the antenna gain will be reduced and the mutual coupling between antennas will be caused. At the same time, higher requirements will be put forward for the measurement accuracy of interferometer. In addition, the multi-baseline fuzzy resolution method has the problem of a large amount of computation due to the need to carry out multidimensional integer search. On the other hand, it is difficult to develop a phase interferometer with a long baseline [9-30].

In this chapter, the fuzzy - free phase difference direction finding method based on short baseline array is introduced and three different methods are presented.

Firstly, the virtual short baseline direction finding method based on onedimensional double-base asymmetrical array is studied deeply, and a measurement method like symmetric double-base array to eliminate phase ambiguity is directly presented.

Then, a direction-finding method based on orthogonal array is given by using the characteristic that the curve shape of the path difference function per unit wavelength is very similar to the cosine function.

Finally, a method of direction finding without phase ambiguity is discussed by integrating the Doppler measurement information to calculate the number of wavelengths contained in the radial distance.

At the same time, this chapter also uses the virtual baseline direction finding method to analyze the influence of the measurement error of wavelength number difference on direction finding accuracy and concludes that the influence of the measurement error of wavelength number on direction finding is extremely limited.

### 5.2. UNAMBIGUOUS PHASE DIFFERENCE DIRECTION FINDING BASED ON VIRTUAL SHORT BASELINE

### 5.2.1. Overview

According to the existing theoretical analysis, the unambiguous direction finding based on a virtual baseline can be realized by using the phase difference obtained by two baselines with unequal lengths, and the difference between the two lengths is less than half a wavelength [8-12]. However, in the analysis process of the existing virtual baseline method, the phase jump problem in the virtual short baseline is hardly noticed [13]. Moreover, such problems are likely to be mainly caused by noise, interference, and other factors.

The analysis in this section confirms that, although the difference between the two baselines is less than half a wavelength, the difference value of the number of wavelengths will not be zero in the direction of the partial angle of arrival, but there is a jump in the case of a large difference between the two radial distances. This shows that phase jump is not a phenomenon only caused by noise and interference.

In this section, the problem that the phase fuzzy value is not zero in the virtual short baseline is analyzed in detail, and a simple correction method is given. Like the unambiguous detection analysis of the change rate of the phase difference, the correction can be realized by discriminating the sine value of the arrival angle, and the correction factor is the scale factor of the difference between the two baselines.

## CHAPTER 6

# Unambiguous Phase Difference Ranging Based on Short Baseline Array 


#### Abstract

This chapter presents several methods of distance ranging for basic short baseline arrays. These methods are either based on basic array positioning equations or on basic physical definitions, but the final mathematical form of these methods is proved to be the same. From the derivation of the ranging solution, we can see that the one-dimensional double-base array is the basis of the plane positioning analysis. Whether it is based on the Doppler frequency shift signal or the motion parameters, it can be attributed to the positioning solution of the one-dimensional double-base array.


Keywords: Ranging, Short baseline arrays, One-dimensional double-base array, Doppler frequency shift, Motion parameters, Direction finding, Doppler rate of change, Radial acceleration.

### 6.1. INTRODUCTION

Short baseline distance ranging has important engineering application requirements. It is the foundation of realizing single station passive ranging technology. The key of the single passive location system is to realize the single station passive ranging.

This chapter presents several methods of distance ranging for basic short baseline arrays. These methods are either based on basic array positioning equations or basic physical definitions, but the final mathematical form of these methods is proved to be the same.

### 6.2. UNAMBIGUOUS PHASE DIFFERENCE RANGING FOR SHORT BASELINE DOUBLE-BASE SYMMETRIC ARRAY

### 6.2.1. Fundamental Formula

Once the unambiguous solution (3.5-2/3) of the path difference is substituted into the ranging formula of the one-dimensional double-base symmetric array:

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$$
\begin{equation*}
r_{2}=\frac{2 d^{2}-\Delta r_{12}^{2}-\Delta r_{23}^{2}}{2\left(\Delta r_{12}-\Delta r_{23}\right)} \tag{1.5-12}
\end{equation*}
$$

We can obtain a seemingly concise form of ranging based on phase difference measurements:

$$
\begin{align*}
r_{2} & =\frac{2 d^{2}-\left(d \sin \theta+0.5 \lambda \Delta^{2} r_{\lambda}\right)^{2}-\left(d \sin \theta-0.5 \lambda \Delta^{2} r_{\lambda}\right)^{2}}{2\left(d \sin \theta+0.5 \lambda \Delta^{2} r_{\lambda}-d \sin \theta+0.5 \lambda \Delta^{2} r_{\lambda}\right)} \\
& =\frac{2 d^{2} \cos ^{2} \theta-0.5 \lambda^{2}\left(\Delta^{2} r_{\lambda}\right)^{2}}{2 \lambda \Delta^{2} r_{\lambda}}  \tag{6.2-1}\\
& =\frac{d^{2}}{\lambda \Delta^{2} r_{\lambda}} \cos ^{2} \theta-0.25 \lambda \Delta^{2} r_{\lambda}
\end{align*}
$$

### 6.2.2. Approximate Solution

If the last term of Equation (6.2-1) is ignored, then:
$r_{2}=\frac{d^{2}}{\lambda \Delta^{2} r_{\lambda}} \cos ^{2} \theta$
The difference function of the path difference per unit wavelength length in the denominator is expressed directly by the difference of path difference:

$$
\begin{equation*}
\Delta^{2} r_{\lambda}=\frac{\Delta r_{i}-\Delta r_{i+1}}{\lambda} \tag{3.3-3}
\end{equation*}
$$

At the same time, the representation of the geometric projection in the right triangle on the numerator is changed to the representation of the difference between two squares in the right edge:

$$
d^{2} \cos ^{2} \theta=d^{2}-\Delta r_{23}^{2}
$$

So there are:

$$
\begin{equation*}
r_{2}=\frac{d^{2}}{\lambda \Delta^{2} r_{\lambda}} \cos ^{2} \theta=\frac{d^{2}-\Delta r_{23}^{2}}{\Delta r_{12}-\Delta r_{23}} \tag{6.2-3}
\end{equation*}
$$

The results obtained are the same as those obtained by approximate simplification analysis in the third section of the second chapter. For one-dimensional doublebase ranging formula (1.5-12), if the higher-order term of two adjacent path differences in the numerator of the equation is approximately considered equal, that is: $\Delta r_{12}^{2} \approx \Delta r_{23}^{2}$, then the ranging solution can be simplified as:

$$
\begin{equation*}
r_{2} \approx \frac{d^{2}-\Delta r_{23}^{2}}{\Delta r_{12}-\Delta r_{23}} \tag{2.3-11}
\end{equation*}
$$

### 6.2.3. A More Regular Representation

The simulation results show that more accurate results can be obtained if the path difference term on the approximate range solution on the molecule is replaced by the product of two adjacent path differences:

$$
\begin{equation*}
r_{2}=\frac{d^{2}-\Delta r_{23}^{2}}{\Delta r_{12}-\Delta r_{23}}=\frac{d^{2}-\Delta r_{12} \Delta r_{23}}{\Delta r_{12}-\Delta r_{23}} \tag{6.2-4}
\end{equation*}
$$

Thus, the expression of the formula is extremely symmetric.

### 6.3. RANGING BASED ON PHASE CHANGE RATE

For a single basis phase difference direction-finding formula:

$$
\begin{equation*}
\sin \theta \cong \frac{\Delta r}{d}=\frac{\lambda}{d}\left(\Delta n+\frac{\Delta \phi}{2 \pi}\right) \tag{3.2-6}
\end{equation*}
$$

By directly differentiating it, we get:

$$
\begin{equation*}
\omega \cos \theta=\frac{\lambda}{2 \pi d} \frac{\partial \Delta \phi}{\partial t} \tag{6.3-1}
\end{equation*}
$$

## CHAPTER 7

## Passive Detection Method Based on Angle Measurement


#### Abstract

This chapter first studies the relationship between angle measurement and phase difference measurement. On the one hand, this helps to understand the physical properties, and on the other hand, it helps to simplify the process of error analysis. Then, two kinds of direction-finding methods which can improve the positioning accuracy are presented. One is a double - station cross - directionfinding algorithm involving higher - order geometric parameters. The other is to use the algorithm characteristic of adjacent path difference to virtually expand the baseline length of the detection array, to improve the accuracy of the bi-station direction-finding system. Whereafter, the passive ranging formula for two detection platforms with different motion directions is researched. The analytical formula of target distance based on angle measurement of two carriers is directly derived in different directions and at different speeds in planar polar coordinate system. In the end. Several methods of the azimuth-only estimation of the course of a moving target in a straight line is presented. Its analytical process has nothing to do with the detection of time, and the analytical formula obtained is purely related to the azimuth angle.


Keywords: Direction-finding, Angle measurement, Fixed single station, Triangulation location, Variable transformation, Virtual extension, angle recursion, Virtual path difference, phase interference, co-location, Range, Airborne passive positioning, Course angle, Sports radiation source, Passive location.

### 7.1. INTRODUCTION

This chapter studies the passive positioning method based on angle measurement. Angle and phase difference are the most basic measurement parameters, and the angle parameters can be obtained by using phase different measurement systems. It is stipulated in this monograph that the angle measurements covered in this book are obtained by means of phase difference measurements.

For real targets, in general, the direction angle changes slowly and has a small range, which is one of the most reliable radiation source parameters. Especially in the modern dense and complex signal environment, the direction parameter is almost the only reliable radiation source parameter. And the requirement of the
timing system between the reconnaissance platforms is low when the orientation angle is used. Therefore, it is of great significance to continue the research of positioning method based on orientation measurement [1-5].

In the first section of this chapter, the relationship between angle and phase difference measurements is studied, which helps to understand the physical properties and, on the other hand, to simplify the analysis during the mathematical analysis of error properties.

The positioning method based on angle measurement still has the disadvantage of low positioning accuracy in engineering implementation, and the existing main approach is to adopt some algorithms to deal with it [6-13]. In the third and fourth sections of this chapter, how to improve the accuracy of direction finding is discussed.

In the third section of this chapter, a two-station cross-direction finding algorithm related to higher order geometric parameters is presented. Different from the existing direction-finding cross-location algorithm, which is obtained directly from the plane geometric function, the geometric projection method is used to transform the expression of path difference into a function based only on the angle measurement, thus a two-station ranging algorithm which is related to the higher order geometric parameters is obtained. The new method has better ranging accuracy, requiring only a few kilometers of baseline to make location measurements for targets up to 300 kilometers away. And it is easy to be extended and applied to single motion station.

The fourth section of this chapter studies how to use the arithmetic features of the adjacent path difference to virtually expand the baseline length of the detection array to improve the accuracy of the measured two-station direction finding system. Firstly, by using the median tangent relationship, it is shown that the expansion based on angle alone cannot improve the positioning accuracy of the two-station direction-finding system. Then, the method of virtual extension of baseline length based on the arithmetic features between adjacent paths is described. Based on this, a method of double virtual baselines extension is presented by using the tangent median relation and the arithmetic property simultaneously.

In the fifth section of this chapter, the analytical formula of target distance based on angle measurement is directly derived in the planar polar coordinate system under different directions and different velocities of double carrier.

Section 6 of this chapter presents three method of azimuth-only estimation of the course of a target moving uniformly in a straight line based on isometric measurement. It has been proved that the course of a target moving at a constant speed can be estimated by at least three azimuth-only measurements in a single stationary station on a two-dimensional plane. However, in fact, the existing analysis process contains time parameters, so the information required to obtain does not seem to be purely directional. This section of the analysis show that on the two-dimensional plane, for the target of uniform linear motion, if the assumption that a single stationary station can obtain the orientation information of target on the route to the target by taking three measurements at equal intervals in succession, the analytical expression of the target course can be derived by adopting pure geometry calculation method, in the case of completely independent of the detection of time parameters.

### 7.2. ERROR CONVERSION FACTOR BETWEEN ANGLE AND PHASE MEASUREMENTS

### 7.2.1. Overview

The phase difference direction finding method based on one-dimensional linear array is essentially a triangulation method, which is directly related to sine and cosine functions. And unfortunately, the measurement accuracy is inversely proportional to the trigonometric function. Therefore, the phase difference direction finding based on one-dimensional linear array has a blind spot in the measurement accuracy.

In this section, the total differential analysis method of ranging error is used to illustrate that the direction finding based on phase difference measurement will have an additional factor in the calculation formula of the accuracy of the location measurement compared with that based on angle measurement only. This reveals the difference between the ranging error based on the angle measurement and the ranging error based on the phase difference detection.

# Passive Detection Method Based on Phasefrequency Conversion 


#### Abstract

On the basis of the existing research, the passive detection method based on Doppler shift is further converted to the passive detection method based on the non-fuzzy phase difference measurement. Based on the phase-frequency conversion, the passive detection methods of two different models, that is a method of detecting a fixed target by a moving station, are presented in this chapter for improving the ranging accuracy. The first one studies the single station passive ranging method which takes the motion trajectory as the baseline of the detection array. The second one studies the ranging method which transforms the single base array into the virtual double base array by the relation between frequency shift and path difference. The research in this chapter provides some reference information on how to improve the performance of single or dual station passive positioning. This chapter proves that the accuracy of the phase difference ranging is better than that of the Doppler frequency difference. The significance of phase-frequency conversion lies in the first use of Doppler positioning method to give the location method which is difficult to obtain by using the phase difference equation, and then through phase-frequency conversion, further obtain better measurement accuracy than Doppler frequency difference measurement.


Keywords: Motion trajectory, Phase-frequency conversion, Single station passive ranging, Doppler rate, Short baseline array, Virtual double-basis linear array, Airborne passive positioning, DF, Frequency difference, Path difference, Doppler shift.

### 8.1. INTRODUCTION

In this chapter, based on the existing research, the passive detection method based on Doppler shift is further converted to the passive detection method based on the non-fuzzy phase difference measurement. Because the present research results are only applicable to achieve unambiguity phase difference detection of Doppler shift in the same receiver, the unambiguity phase difference measurement of Doppler shift still has some limitations. At present, the method based on phase-frequency

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conversion can only be used to analyze the passive detection of fixed targets by moving stations.

The feature of positioning technology based on path difference is that the measurement accuracy is proportional to the length of the baseline, so positioning technology based only on short baseline is generally impossible to achieve high positioning accuracy. Based on the phase-frequency conversion, the passive detection methods of two different models that is a method of detecting a fixed target by a moving station, are presented in this chapter from the requirements of how to improve the ranging accuracy. The first one studies the single station passive ranging method which takes the motion trajectory as the baseline of the detection array. The second one studies the ranging method which transforms the single base array into the virtual double base array by the relation between frequency shift and path difference.

Compared with the multi-station positioning system, the mobile single-station positioning system does not need a large amount of communication data transmission and has the advantages of simple structure and flexible equipment. Therefore, it has a broad application prospect in many civilian and military fields such as navigation, aviation, satellite positioning early warning, electronic reconnaissance, and guided anti-radiation weapons [1-2]. The existing measurement methods for airborne single station passive detection mainly include multi-point direction finding, Doppler shift, and so on. These methods usually require continuous multi-point detection [3-8]. There are also several theoretical studies that provide a method to realize positioning with only one detection [9].

Although the single-station system has advantages such as good independence and flexibility and can guarantee the completion of positioning and tracking tasks in stealth, it is more difficult to implement because the information obtained by a single station is relatively less than that obtained by multiple observation stations. Facing the development of modern high technology, some traditional airborne single station positioning methods, such as the direction finding positioning method, may not have any advantages at all. On the other hand, various new airborne single-station positioning technologies currently under development, such as the positioning method based on the phase difference change rate and the positioning method based on the Doppler change rate, etc. [10-16], are also limited by the technical measurement methods. That's because the rate of change itself is all about the time derivative. In addition, in the case of airborne single station with
short baseline application, it is relatively difficult to detect the change rate of phase difference or Doppler frequency shift under the requirement of real-time because the magnitude is very small, and it is also difficult to obtain better measurement accuracy [17-20].

The research in this chapter provides some reference information on how to improve the performance of single or dual station passive positioning.

### 8.2. SINGLE STATION PASSIVE RANGING METHOD BASED ON MOTION TRAJECTORY

### 8.2.1. Overview

Assuming that the moving single station moves along a straight line, a single base detection array can be constructed virtually by using the motion track of the station itself. On this basis, the basic ranging solution is given by using Doppler rate ranging formula. Then by phase-frequency conversion, the passive ranging solution of moving single station based on unambiguous phase difference measurement can be obtained.

### 8.2.2. Basic Framework

As shown in Fig. (8.2-1), it is assumed that a Doppler receiver is installed on a moving single station. The detection platform measures the Doppler frequency shift of the target at the detection point $A$ and obtains $f_{d 1}$. Then, it moves along a straight line to the detection point $B$ for detection and obtains Doppler frequency shift $f_{d 2}$

Fig. (8.2-2) shows the short baseline antenna array configuration for the moving single-station detection platform. It refers to Fig. (5.2-10). Where $E_{1}, E_{2}$ and $E_{3}$ are virtual short baseline arrays for direction finding. $E_{4}, E_{1}$ and $E_{3}$ form a doublebase symmetric array for phase detection. $E_{1}$ is set as the reference point of positioning measurement. In engineering application, only three antenna units can be used to achieve direction finding and ranging functions. The present layout can mainly make the datum center of direction finding coincide with the datum center of ranging, thus reducing the complexity of calculation.

## CHAPTER 9

# Passive Positioning Method, Based on Combined Measurement, of Fixed Two Stations 


#### Abstract

In this chapter, two passive detection methods of long baseline fixed bistatic are presented based on the short baseline composite array. The first method is a two-station phase differential direction method with a long baseline. Direction finding and ranging are realized at each station using short baseline phase difference measurement technology. The path difference between two stations is obtained by using the radial distance measured by the two stations, respectively. On this basis, using the midpoint direction finding solution of a single basis, the phase differential direction finding formula of two stations with long baseline can be given directly. The second method is a method that uses a similar recursive method to implement the long baseline multi-station passive detection based on phase difference measurement without ambiguity. The similar recursion method is a method to obtain the long base path difference by using the similarity of triangles according to the path difference of short base line array, and then construct the virtual three-station linear array, from which the location calculation can be carried out directly by using the ranging formula of the double base array. It may be a feasible method to construct a long baseline multi-station phase difference detection system using a short baseline composite array.


Keywords: Combination measurement, Long baseline positioning, Short baseline, Radial distance, Dual-station location, Phase difference direction finding, Similar recursion, Virtual array, Baseline extension, Multi-station location, Passive location.

### 9.1. INTRODUCTION

Under the current technical conditions, multi-station passive positioning technology is still a more effective remote passive detection and positioning method [1-6]. In terms of physical properties, the error measurement equation of phase difference positioning is not only independent of the large value of the speed of light but also proportional to $\lambda / 2 \pi$. Therefore, compared with multi-station time difference positioning technology, multi-station phase difference positioning can obtain better positioning accuracy. However, one of the main obstacles to the

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existing phase difference measurement techniques to be applied to the passive positioning of long baselines is that the phase ambiguity can be effectively solved, and high phase measurement accuracy can be obtained, only at short baselines of a few dozen wavelengths in length. According to the existing positioning methods based on path difference measurement, all have the characteristic that the baseline is proportional to the measurement accuracy. It seems that it is impossible to directly realize the remote accurate positioning by using the short baseline length of only dozens of wavelengths.

Another major obstacle to the direct use of long baseline passive positioning is the difficulty in ensuring the accuracy of the phase difference measurement between arrays. One difficulty to overcome is that due to the sensitivity of the phase difference measurement in the microwave band, a small change in the length of the baseline may lead to large measurement errors. Therefore, the attempt to directly realize the long baseline phase difference positioning may not meet the practical conditions of the present engineering.

It may be a feasible method to construct a long baseline multi-station phase difference detection system using a short baseline composite array. In chapter 8, based on the Doppler frequency shift ranging method, and using the phase frequency transformation, a method to realize the long baseline passive phase difference location on the moving platform by using the short baseline composite array is presented. In this chapter, two passive detection methods of long baseline fixed bistatic are presented based on the short baseline composite array.

### 9.2. A LONG-BASELINE DUAL-STATION DIRECTION FINDING METHOD

### 9.2.1. Overview

In this section, a method of dual station phase difference direction finding with long baseline is presented by adopting the combination measurement method. Direction finding and ranging are realized at each station using short baseline phase difference measurement technology. Firstly, the path difference between two stations is obtained by using the radial distance measured by the two stations, respectively. On this basis, using the midpoint direction finding solution of a single basis, the phase differential direction finding formula of two station with a long baseline can be given directly.

### 9.2.2. Basic Layout

As shown in Fig. (9.2-1), there are two fixed detection stations $A$ and $B$ with a distance of $D$. In each station, a short baseline double-base array is installed, and phase difference measurement technology is used to realize direction finding and ranging.


Fig. (9.2-1). Composite array diagram.
The single-station short baseline array uses the same configuration as that presented in Section 2 of Chapter 8. In theory, the virtual short baseline direction finding technique can be used to solve the target arrival angles at two stations. However, to simplify the analysis, this section directly assumes that the direction-finding method based on phase difference measurement has been used to obtain the angle of arrival of the target and to directly analyze the error of the angle.

### 9.2.3. Ranging Algorithm

According to the formula (6.6-1) in Section 6 of Chapter 6.

$$
\begin{equation*}
r_{2}=\frac{d^{2} \cos ^{2} \theta_{2}}{\lambda \Delta^{2} r_{\lambda}}+0.25 \lambda\left(\Delta^{2} r_{\lambda}\right) \tag{6.6-1}
\end{equation*}
$$

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