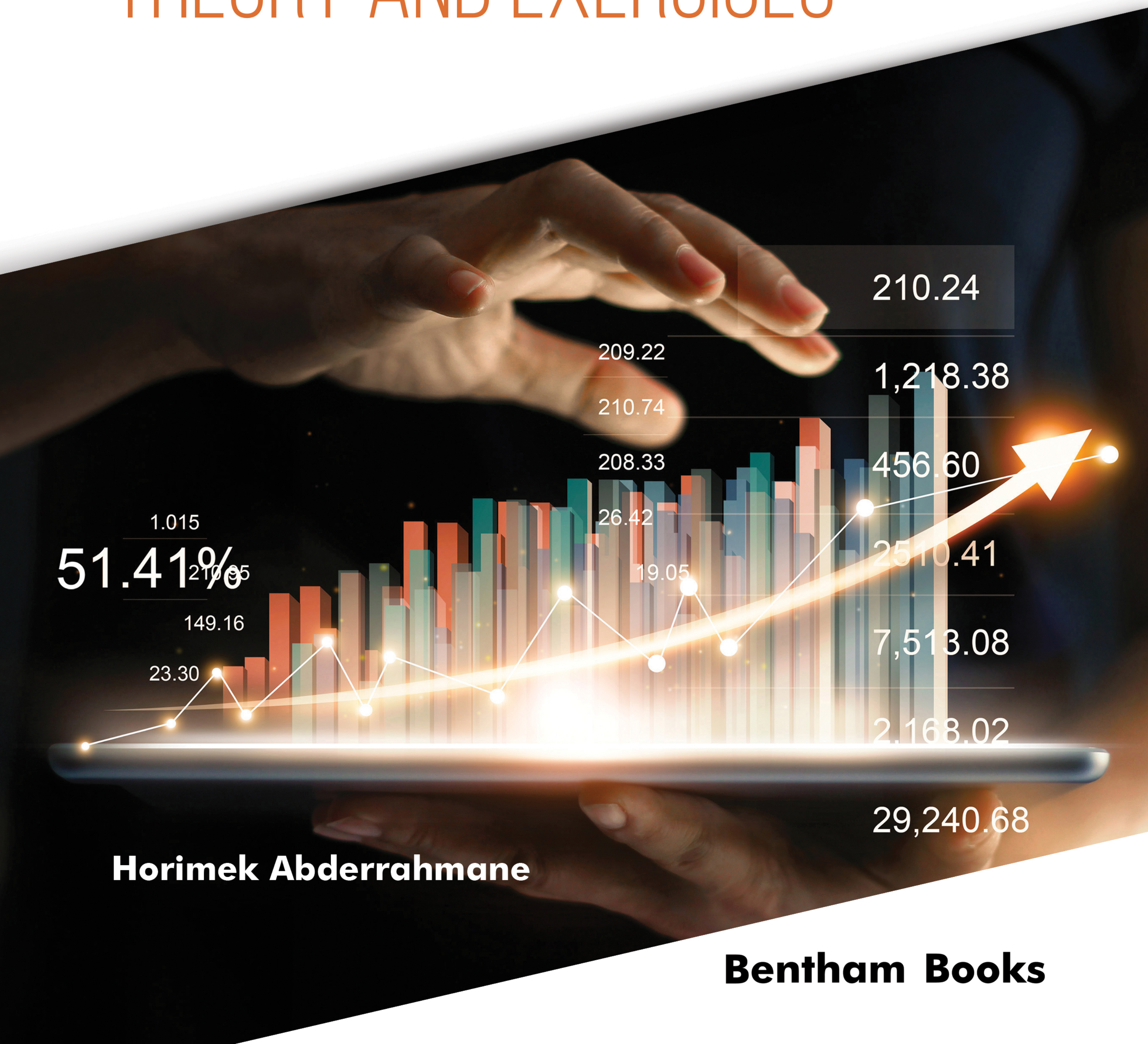
PROBABILITY AND STATISTICS: THEORY AND EXERCISES



Horimek Abderrahmane

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Probability and Statistics: Theory and Exercises

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FOREWORD

The book “Probability and Statistics: Theory and Exercises” by Dr. Horimek Abderrahman, is an excellent scientific contribution, written in a simple, clear English language. Presented in a practical, user-friendly, and relevant manner, this book will enable engineering and science students and teachers to understand the concepts and theories of probability and statistics and apply them in real-life situations. This is a real contribution to spreading the culture of statistical thinking among the Algerian higher education community and business leaders in the country.

Statistical thinking has been adopted by world-class organizations as a way of understanding our complex world by describing it in relatively simple figures that capture essential aspects of its structure and components. Statistical thinking, through probability and statistics, provides us with evidence about our past and present and allows us to discover insights into the future. Through statistics, evidence-based decision-making processes can be established, so higher product or service quality can be delivered to customers; hence contributing to an improved Quality of Life. Indeed, statistics have been used to predict future events, and foresight even the future of nations and citizens, hence contributing to build a sustainable future.

As statisticians, we always refer to the famous statement from H.G. Wells, “Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.”, I would rather say that this event is already happening, and we need to dig deeper in this science through this book and other reference textbooks.

It is expected that this manuscript will be a useful guide for all individuals who are involved in studying events and phenomena related to social, technical, engineering, or medical sciences.

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PREFACE

We all know the importance of probability and statistical calculations in everyday life. The trend toward perfect precision requires not making errors; unfortunately, the error is human. Consequently; researchers are majorly interested in reducing this error, by means of the mastery of the world of chance which is really not as random as it seemed 500 years ago. Currently, we can anticipate events while knowing the past and the laws of evolution thanks to the laws of probability and data censuses. Therefore, we can learn the basics of probability and statistics has become indispensable to specialists in mathematics and economics, as well as to field engineers who have a large part of their duties based on statistical analyses.

This course is intended primarily for students of technical sciences at the undergraduate level; but it is also beneficial to those enrolled in master's program and even to those who have already graduated. Without a doubt, this course offers considerable help to the teachers, with numerous illustrative examples and exercises solved in detail with discussions.

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ACKNOWLEDGMENT

I would like to thank all those who have posted detailed lessons and exercises on the subject of "probability and statistics" on the internet. I would like to especially thank Professor **Mohamed AICHOUNI** who honored me by writing the Forward page after consulting this book despite his other commitments.

PART 1: PROBABILITIES

CHAPTER 1

Reminder on the Theory of Sets

Abstract: Accidental events in chemical and process industries may have catastrophic consequences. The present chapter aims to discuss the hazards and risks in chemical and process industries, where chemical species are used and/or transformed. After defining the concept of chemical risk, the possible accidental events in the process industry are presented based on their probability of occurrence. Some examples of relevant chemical accidents that occurred in the past are thoroughly discussed further. Safety measures *(i.e.,* preventive and protective procedures) in safety and process industries and primary and secondary reactions are also described. Finally, a screening method capable of providing a hazard evaluation by calculating the power released during the thermal decomposition of a substance *(i.e.,* the CHETAH method) is presented.

Keywords: Accidents, Chetah method, Probability, Primary and secondary reactions, Risk.

INTRODUCTION

The analysis of specialized books in probability (and statistics) shows that the majority of them begin with a passage, or even an entire chapter on the sets and their relationships, followed by the probability course itself. The idea is that, exactly, the same laws are applied with the difference of nomination, of course in the sense of probability. The probability is defined as the ratio between the number of favorable cases to that of possible ones. In the theory of sets, it is the ratio (or percentage) of a part of the set to the set itself, counting the number of elements in each of them. The same logic for all known laws in sets theory.

For this, we will start with a brief reminder of sets and their main laws, which will serve as a beneficial introduction to the course given the simplicity of manipulations of the laws on sets. Gradually, the definitions used in probability will be introduced in a flexible and clear manner.

DEFINITION

A set is *a collection* of clearly defined objects called elements of that set [1-3].

Examples:

-The set of odd numbers less than 14;

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-All the vowels of the alphabet;

-All the students in our class.

Warning: We cannot say for example: "the set of large numbers" or "the set of intelligent people"…etc, because it is not clear what would be the elements of these "sets" !.

Notations: The sets are denoted by letters, most often in upper case: ***A***, ***B***, ***C***, ***D***, ... The elements of the set are then *listed* in braces **{ }**.

Examples:

A={0 ; 1 ; 2 ; 3 ; 4 ; 5 ; 6} ;

B={1 ; 3 ; 5 ; 7 ; 9 ; 11 ; 13} ;

V={a ; o ; u ; i ; e ; y}

These sets are said to be defined by ***enumeration*** or by ***extension***.

When the number of elements in a set is very large or even infinite, we <u>*cannot*</u> enumerate them all. To define such a set, we give a property of its elements which makes it possible to understand what these elements are: we then say that the set is defined in ***comprehension***.

Examples:

M={integers below 9000} ;

M={***x/x*** is an integer less than 9000}

We read: "***M*** is the set of elements ***x*** such that ***x*** is an integer less than 9000".

Note: Some sets can be defined by enumeration and comprehension.

Example:

G={***x/x*** is an even number between 8 and 17} or ***G***={8 ; 10 ; 12 ; 14 ; 16}

Cardinality of a Set

We call the cardinal of a set ***A***, the number of elements of this set.

Examples

A={8; 10;12; 14; 16 ;18 ;20}

Card (***A***)=7

B={2nd year Mechanical Engineering students}

Card (***B***)=68 (for example)

The Symbols $\in$ and $\notin$

For any set ***E*** and any element *x*:

- If *x* is an element of ***E***, we write $x \in E$ and we read: *« x belongs to **E** »*
- If *x* is not an element of ***E***, we write $x \notin E$ and we read: *« x does not belong to **E**»*

Example: IF ***E***={5; 9; 12} we write : $5 \in E$, $9 \in E$ et $12 \in E$, but $8 \notin E$, $2 \notin E$,… *etc*.

Equal Sets

Two sets are *equal* if they have the same elements.

Example: Which of the following sets are: equal?, Different ?

A={1; 2; 3; 4}

B={1; 4; 5; 7} $A \neq B$ / $B \neq C$ / **A=C**

C={3; 1; 4; 2}

Empty Set

If the set contains no elements. It is noted as $\varnothing$ **.**

Example: ***A***={*x*/ *x* is a student in our class who is over 3m tall}; So: $A=\varnothing$

CHAPTER 2

Introduction to Basic Definitions in Probability

Abstract: In this chapter, the basic definitions of probability theory are presented. The logic of presenting the results as events and their quantification, in order to know those that are highly probable or the reverse, are all detailed. It should be noted that in this chapter, we are only interested in knowing how to determine the value of the probability for a single random experiment.

Keywords: Events, Probability definition, Probability space, Random experiment.

INTRODUCTION

In this chapter, we will present many definitions essential in the calculation of the probability. As already mentioned, calculating a probability comes to calculating the percentage of achievement or non-achievement of a phenomenon (or a result). So, if we determine the total space of possibilities and assume (even without doing) the phenomenon, it remains to determine its results (elements for a set) which can be classified according to our interest (characteristic of the set). The assumption of the phenomenon is called random experience (total set or universe) to be determined, the outcomes of which are called events (elements or subsets).

RANDOM EXPERIMENT (RE)

Definition: A Random Experiment is a renewable experiment, the result of which cannot be predicted, and which, when repeated under identical conditions, does not necessarily give the same result each time it is repeated [3]. Each renewal of the experiment is called a trial (test, throw, *etc*.). A trial can combine several elementary trials, either consecutively or simultaneously. Trials can be more or less independent. The set of possible outcomes, of a random experiment constitutes the base of this experiment.

Example: If you toss a coin, you cannot predict which side it will land on. So: *Throwing a coin is a random experience in the sense of probability*.

Events

Definition: Let Ω be a set with a tribe $\boldsymbol{F} = \mathrm{P}(\Omega)$. The elements of $\boldsymbol{F}$ are called events or issues.

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Elementary Events

Let ***E*** be a *random experiment*, an event ***A*** linked to the experiment ***E***, is said to be an elementary event if it is only realized by a single outcome of this experiment.

Example: We roll a dice (Fig**. 2.1**). The exits are one of the 6 numbers appearing on the upper face after immobilizing the dice. The event "*The number on the top side of the die is 4*" is an elementary event.

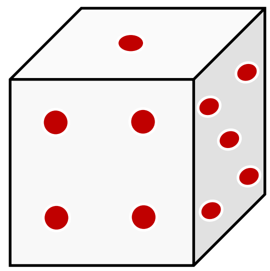

Fig. (2.1). A six-sided dice.

Note: An event can be a set of elementary events, and can be seen as a subset of Ω, the set of all possible elementary events called *fundamental space* or *universe*.

Example: For a dice roll: $\Omega = \ 1\ ;\ 2\ ;\ 3\ ;\ 4\ ;\ 5\ ;\ 6\ = 1;2;3;4;5;6$

Certain Event

Let ***E*** be a random experiment and ***A*** an event linked to this random experiment. We say that ***A*** is a *certain event*, if it is *realized whatever the outcome of the experiment **E***.

Example: We roll a dice. The event: "the number on the top side of the dice is less than 7", is a certain event.

Note: Ω is a certain event and $\varnothing$ is an empty event.

Impossible Event

Let ***E*** be a random experiment and ***A*** is an event. We say that ***A*** is an *impossible event*, if *it is not realized, whatever the outcome of the experiment **E***.

Example: We roll a dice. The event: "the number on the top side of the dice is number 7" is an impossible event.

Complementary Event

Let $\boldsymbol{E}$ be a random experiment and $\boldsymbol{A}$ is an event. We say that $\bar{A}$ (or A^c) is the complementary event of $\boldsymbol{A}$, if it is composed of elementary events not realized by $\boldsymbol{A}$. It is defined as:

$$\overline{A} = \Omega - A \quad \textbf{(2.1)}$$

Example: In the experiment of rolling a dice, if $\boldsymbol{A}$ is the event: "have an even number on the upper face", then: $A = \{1;3;5\}$. The complementary event $\bar{A}$ (not $\boldsymbol{A}$) is: "not to have an even number". Therefore: $\overline{A} = \{2;4;6\}$

Properties:

i. $\overline{\overline{A}} = A$;

ii. $A \cap \overline{A} = \varnothing$;

iii. $A \cup \overline{A} = \Omega$; **(2.2)**

iv. $\overline{\Omega} = \varnothing$;

v. $\overline{\varnothing} = \Omega$.

Incompatible Events

Let $\boldsymbol{E}$ be a random experiment and two events $\boldsymbol{A}$ and $\boldsymbol{B}$ are linked to this random experiment. We say that they are *incompatible* if they *cannot be carried out simultaneously* (Fig. **2.2**).

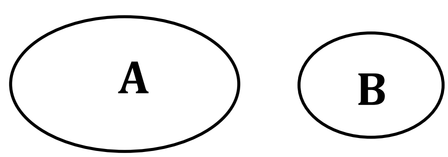

Fig. (2.2). Two incompatible events.

CHAPTER 3

Combinatory Analysis

Abstract: This chapter presents in detail the calculation techniques for random experiments with a high number of outcomes that are generally impossible to calculate classically. The outcomes can be the number of favorable cases or that of possible ones, necessary to calculate the probability value. It is enough to imagine the situation well, to know which law should be used.

Keywords: Arrangements, Combinations, Permutations, Principle of multiplication.

INTRODUCTION

The objective of combinatorial analysis (also known as Combinatorics or Counting Technique), is to learn to count the finite number of elements with large cardinalities.

MULTIPLICATION PRINCIPLE (GENERAL PRINCIPLE OF ENUMERATION)

It allows counting the number of results of experiments which can be broken down into a series (succession) of sub-experiments [1,2,5].

Principle: Assuming that an experiment is the succession of m sub-experiments. If the i^{th} experiment has n_i possible results, for $i = 1, 2, \ldots, m$. So, the total number of possible outcomes of the overall experiment is:

$$\boxed{N = \prod_{i=1}^{m} n_i = n_1 \times n_2 \times \ldots\ldots \times n_m} \qquad \textbf{(3.1)}$$

Example 01: You buy a suitcase with a code formed by 04 digits (from 0 to 9).

- How many ways can you choose your code?

Solution: We can imagine the code to be formed as a random experience, in which, the choice of each digit constructs a sub-experience.

The code can take many formulations, for example:

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0000 / 0001 / 0002 /……….. / 1111 / 1112 / ………. / 9997 / 9998 / 9999

Therefore:

- The **first** digit can be chosen in **10 ways** (from **0** to **9**);
- The **second** digit can be chosen in **10 ways** (from **0** to **9**);
- The **third** digit can be chosen in **10 ways** (from **0** to **9**);
- The **fourth** digit can be chosen in **10 ways** (from **0** to **9**).

The total number of possible codes is therefore: $\mathbf{10\times10\times10\times10=10^4}$ (10000 codes!!!).

The multiplication principle verifies this result. The number of codes is N while the number of possibilities of each digit is n_i. The code is made up of 04 digits, so m=4.

$$N = \prod_{i=1}^{4} digt_i = digt_1 \times digt_2 \times digt_3 \times digt_4 = 10\times10\times10\times10 = 10^4 \qquad \textbf{(3.2)}$$

Example 02: How many license plates can we have, if it is made up of: **02** different letters followed by **03** digits (from **0** to **9**)? The first digit (*on the left*) cannot be a **0**.

Solution: With the specified considerations, the plate has the following shape (examples):

AB100 / AB101 / AB102 /……….. / BA100 / BA101 / ………. ZY998 / ZY999

- The **first letter** can be chosen in **26 ways** (from **A** to **Z**);
- The **second letter** can be chosen in **25 ways** (from **A** to **Z** *except* the **first letter** chosen);
- The **first digit** can be chosen in **9 ways** (from **1** to **9**);
- The **second digit** can be chosen in **10 ways** (from **0** to **9**);
- The **third digit** can be chosen in **10 ways** (from **0** to **9**).

So the plates' number for our considerations is N. m equal to 5 (plate with 5 characters) and n_i changes depending on the character:

$$N = \prod_{i=1}^{4} charac_i = lett_1 \times lett_2 \times digt_1 \times digt_2 \times digt_3 = 26\times25\times9\times10\times10 = 585000 \qquad \textbf{(3.3)}$$

Important note: All that we will see in what follows are only special cases of the principle of multiplication. We must always try to make the connection in order to be able to resolve very complex cases.

PERMUTATIONS

There are two cases, permutations without repetition and those with repetition [1-2].

Permutation without Repetition

A permutation without repetition of n distinct (different) elements $e_1, e_2, \ldots, e_n$, is an ordered rearrangement (the order is respected) without repetition of one or more elements of the n elements.

Example: We have 02 books. The 1st of Statistics (**S**) and the 2nd of Mathematics (**M**). We want to put them on a shelf.

- How many ways can we do this?

Solution: We will schematize the situation to better understand (Fig. **3.1**):

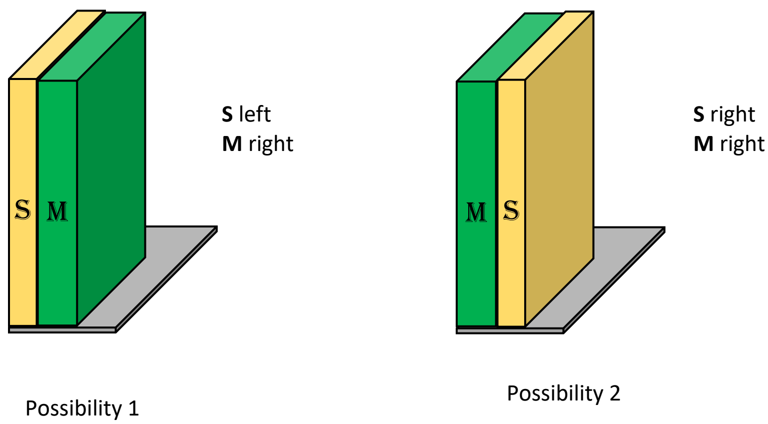


Fig. (3.1). Permutations for two elements.

For the two books we have two possibilities. Either the **S** on the left (starting direction) then **M** must necessarily be on the right, or the opposite. In reality, the space occupied by the two books is the same; we only swapped the positions (positions' permutation) between them.

CHAPTER 4

Calculation of Probabilities

Abstract: After learning how to determine the probability value for a single random experiment, this chapter gives you the tools for compound random experiments. The probability tree that graphically presents the experiment is defined first, followed by numerous details and laws, leading to the mastery of calculation tricks for an experiment composed of n sub-experiments.

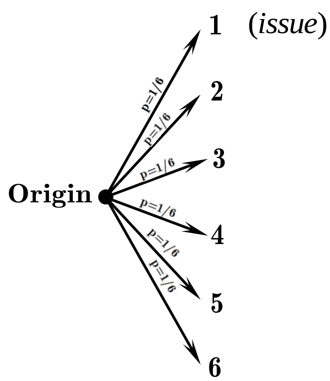
Keywords: Bayes' theorem, Conditional probability, Global probability, Total probability law, Tree diagram of probabilities.

INTRODUCTION

After having defined in detail what is a probability (Laplace's law), and how to calculate it for simple cases and those with big cardinality, in this chapter, we move on to describing its calculation for complex cases with exploitation of the definitions seen on the events.

PROBABILITY TREE

Let consider a random experience of rolling a non-rigged dice. Experience' outcomes can be represented by the following tree diagram of probabilities (Fig. **4.1**) [1]:



In this case, the dice is not rigged. So, all faces have the same chance of appearing. *i.e*. that we have equiprobability. The probability for such an issue is:

$$\boldsymbol{p} = 1/\boldsymbol{n} = 1/6 \;\; (\boldsymbol{n} = 6 : \textbf{\textit{Nbr of faces}})$$

Fig (4.1). Probability tree for rolling a fair six-sided dice.

Compatible Events

Two events $\boldsymbol{A}$ and $\boldsymbol{B}$ are compatible, if the realization of $\boldsymbol{A}$ does not interfere (does not prevent) the realization of $\boldsymbol{B}$ and *vice versa*.

Example

$\boldsymbol{A}$= « Have an **odd** number on the six-sided dice» $\Rightarrow \boldsymbol{A} = \{1; 3; 5\}$ and $\boldsymbol{p}(\boldsymbol{A}) = 3/6 = 1/2$

$\boldsymbol{B}$= « Have a **multiple** of **3** the six-sided dice» $\Rightarrow \boldsymbol{B} = \{3; 6\}$ and $\boldsymbol{p}(\boldsymbol{B}) = 2/6 = 1/3$

For the event, $\boldsymbol{A} \cup \boldsymbol{B} = \{1; 3; 5; 6\}$ the probability will be: $\boldsymbol{p}(\boldsymbol{A} \cup \boldsymbol{B}) = 4/6 = 2/3$

We can easily see that issue **3** exists in both events $\boldsymbol{A}$ and $\boldsymbol{B}$, so they are compatible. In addition, the probability of their union amounts to summing their probabilities independently and then subtracting that of the common outcome to be counted only once (see Chapter 2).

So, for every two compatible events, we have:

$$\boxed{\boldsymbol{p}(\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{p}(\boldsymbol{A}) + \boldsymbol{p}(\boldsymbol{B}) - \boldsymbol{p}(\boldsymbol{A} \cap \boldsymbol{B})} \quad \textbf{(4.1)}$$

Incompatible Events

Two events $\boldsymbol{A}$ and $\boldsymbol{B}$ are incompatible, if their simultaneous realization is impossible.

Example

$\boldsymbol{A}$= « Have an **odd** number on the six-sided dice» $\Rightarrow \boldsymbol{A} = \{1; 3; 5\}$ and $\boldsymbol{p}(\boldsymbol{A}) = 3/6 = 1/2$

$\boldsymbol{B}$= « Have a **multiple** of **2** the six-sided dice» $\Rightarrow \boldsymbol{B} = \{2; 4; 6\}$ and $\boldsymbol{p}(\boldsymbol{B}) = 3/6 = 1/2$

For the event $\boldsymbol{A} \cup \boldsymbol{B} = \{1; 2; 3; 4; 5; 6\}$ the probability will be: $\boldsymbol{p}(\boldsymbol{A} \cup \boldsymbol{B}) = 6/6 = 1$

Given the incompatibility of events, there are no common issues. The probability of union is therefore the direct summation of their probabilities.

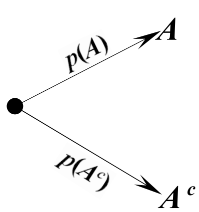
We have: $A \cap B = \varnothing \quad \Rightarrow \quad p(A \cap B) = 0$

Thus, we have the law:

$$\boxed{p(A \cup B) = p(A) + p(B)} \qquad \textbf{(4.2)}$$

Note: The laws of probability calculus for two compatible or incompatible events can be generalized to ***n*** events.

Let's take back our first example (throwing the dice). The weighted tree diagram (Fig. **4.2**) can be put in the following general form (for any considered event):



A may be defined in more than one way and named differently.

If we define: $A = \{1;\, 3;\, 5\}$

$A^c = \Omega / A = \{2;\, 4;\, 6\}$ #

$p(A) = \dfrac{3}{6} \quad \Rightarrow \quad p(A^c) = 1 - p(A) = \dfrac{3}{6}$

Note : $A \cap A^c = \varnothing$; $p(\Omega) = 1$

Fig. (4.2). Generalized form of the probability tree for a RE.

If we roll the dice twice in succession, we can define the events $A = \{1;\, 3;\, 5\}$ on the 1st launch and $B = \{3;\, 6\}$ on the 2nd launch. The probability tree diagram will therefore be composed of two small trees, whose origin of the second is the end of the first regardless of the outcome. For this, we obtain the following tree diagram (Fig. **4.3**):

CALCULATION RULE

Starting from a node, we realize the partition of a sub-universe. Here, for example, the roll of the 2nd dice is a sub-universe of the random experience made up of the two rolls [1-2].

Rule 01: The sum of the probabilities of all branches starting from the same node equals **1.**

$$p(A) + p(A^c) = \frac{1}{2} + \frac{1}{2} = 1 \quad //// \quad p(B) + p(B^c) = \frac{1}{3} + \frac{2}{3} = 1$$

CHAPTER 5

Random Variable

Abstract: After the great progress in the theory of probability as a science, the passage towards more flexible and representative definitions and the possibility to be implemented numerically gave the need for a new formulation. From there, the random variable came into existence. Although random problems can be of different kinds, the random variable can be discrete or continuous, qualitative or quantitative. This chapter is dedicated to its extended definition and its exploitation in a simplified way.

Keywords: Bernoulli's random variable, Probability law, Quantitative and qualitative random variables, Random variable, Types of random variables.

INTRODUCTION

In order to make probability calculations more practical, with the possibility of integrating them into calculation codes, the random variable was introduced. In a simple way, we can imagine this as the search for a representative function $f(x)$ of the outcomes of a random experiment, in which x is the random variable. More details will be given in the subsequent sections.

RANDOM VARIABLE DEFINITION

We will start with two examples in order to define what is a random variable (**RV**)?

Let's consider the random experience of throwing a two-sided token numbered 1 and 2 and a six-sided dice (from 1 to 6) [2, 3, 6].

The universe Ω of this experience is therefore couplets (n=2×6=12):

$$\boldsymbol{\Omega = \{(1;1);(1;2);(1;3);(1;4);(1;5);(1;6);(2;1);(2;2);(2;3);(2;4);(2;5);(2;6)\}}$$

From the universe Ω, we can define infinity of random variables. Consider two illustrative examples.

Example 01: The random variable $\boldsymbol{x}$ attached to each outcome, is equal to the *sum of the digits* of the outcome (*i.e*. the sum of the digit on the token and the one on the dice). $\boldsymbol{x}$ therefore will take seven (07) possible values:

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$$x = 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8$$

Example 02: The random variable ***x*** attached to each outcome is equal to the number of *even digits* in the outcome. ***x*** will therefore take three (03) possible values:

$$x = 0 ; 1 ; 2$$

The **0** here is a value of ***x***. It means that we had a result with odd digits only, for example the couplet $(1 ; 5)$. It should not be forgotten therefore (*i.e.* x=**0**).

Therefore, A random variable is a function, denoted by ***x*** (usually), defined on Ω and has its values in $\Re$.

$$\Omega \xrightarrow{\quad x \quad} \Re \qquad \textbf{(5.1)}$$

For example 01 : $(a ; b) \xrightarrow{x} a+b$

For example 02 : $(a ; b) \xrightarrow{x} a \ even\, x=1 ; b \ even \ x=1 \ (or \ x=0)$

Sets Defined Using *x*

Let ***x*** be a random variable defined on Ω. The possible values of ***x*** are noted $\boldsymbol{x_i}$ and we can write: $\Omega \longrightarrow \{x_1 ; x_1 ; ..... ; x_p\}$.

The set of antecedents of $\boldsymbol{x_i}$ by ***x*** is a subset of Ω. This, is therefore, an event that is noted: $[x = x_i] \ \ or \ \ (x = x_i)$.

With **example 01:** $[x = 4]$:The sum of the digits equal to **4** is achieved by *two couplets* which compose the event $\{(1 ;3); (2 ; 2)\}$.

We write: $[x = 4] = \{(1 ;3); (2 ; 2)\}$

We can also define inequalities on random variables.

$[x < 4]$: The sum of the digits is less than **4**.

This definition requires the use of knowledge already acquired during previous courses.

We can write: $[x<4]=[x=2]\cup[x=3]$

Therefore: $[x<4]=\{(1;1)\}\cup\{(1;2);(2;1)\}=\{(1;1);(1;2);(2;1)\}$

Probability Law of a Random Variable *x*

For each value x_i of x, we have a corresponding probability. Writing the values x_i affected by their corresponding probabilities p_i in a table is called the "***probability law of x***" (Table **5.1**). This is the case with a *discrete random variable* of course (spaced x_i).

Table 5.1. General form of the law of x.

x	x_1	x_2	....	x_p	-
$p[x=x_i]$	p_1	p_2	....	p_p	$\sum_{i=1}^{p} p_i = 1$

With **example 01**: $[x=4]=\{(1;3);(2;2)\}$

$$p[x=4]=\frac{card[x=4]}{card(\Omega)}=\frac{card\{(1;3);(2;2)\}}{card(\Omega)}=\frac{2}{12}=\frac{1}{6}$$

Following the same logic to other x_i values, we arrive at the following Table (**5.2**):

Table 5.2. Law of x of Example 1.

x	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$p[x=x_i]$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

With **example 02*:*** $[x=0]=\{(1;1);(1;3);(1;5)\} \quad \Rightarrow \quad p[x=0]=\frac{3}{12}=\frac{1}{4}$

Following the same logic to other x_i values, we obtain the probability law (Table **5.3**):

CHAPTER 6

Distribution Laws

Abstract: In this last chapter of the probability part, the laws of probability are presented. These laws are very important even in statistics. They govern difficult phenomena by simple mathematical formulas, obtained after a few developments. The mastery of these laws will allow the classification of the phenomena by the amounts of the probabilities and the number of elements in the samples (or the population).

Keywords: Binomial Law, Central limit theorem, Distribution function, Normal law, Poisson's Law, Uniform law.

INTRODUCTION

Like any field, having formulas that are simple to use, instead of going through calculation codes - sometimes essential - is well desired. This implies the formulation of laws of variation of the random variable ***x***, often called ***distribution laws***. They are generally valid for certain ranges of variation of ***x***, or for certain characteristics of the random experiment itself.

DISTRIBUTION FUNCTION (CUMULATIVE DISTRIBUTION FUNCTION (CDF))

The distribution function of a random variable ***x*** (or precisely its law), is the function F_x from $\mathbb{R}$ to $[0,1]$, which associates with any value of ***x***, the probability of obtaining a value less than or equal to the value of ***x*** considered [1, 4, 6].

- For a discrete random variable:

$$F_x = p\left(x \le x_i\right) = \sum_{1}^{i} p_i \tag{6.1}$$

We can notice that F_x represents the cumulative probability of ***x***.

- For a continuous random variable:

$$F_x(x) = \int_{-\infty}^{x} f_x(y).dy \tag{6.2}$$

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Where f_x (or $p(x)$)represents the density function of $\boldsymbol{x}$, which describes its law.

Discrete Random Variable

For the case of a *discrete random variable*, the distribution function leads to a ***staircase-shaped*** evolution:

Consider the following distribution law of $\boldsymbol{x}$ Table **6.1**.

Table 6.1. Distribution function calculation.

$\boldsymbol{x}$	-3	0	3	6
$\boldsymbol{P(x=x_i)}$	1/8	3/8	3/8	1/8
$\boldsymbol{F_{xi}}$	1/8	4/8	7/8	1
	$\sum_1^1 p_i = 1/8$	$\sum_1^2 p_i = p_1 + p_2 = 1/8 + 3/8 = 4/8$	$\sum_1^3 p_i$	$\sum_1^4 p_i$

Graphical Representation

The *probability* is represented graphically by *bars*, while the *distribution function* is represented by *horizontal segments* (Fig. **6.1**). Note that for F_x, we have the value **0** and the value **1** *outside* the domain of the experiment (before and after). This can be explained as follows: before $\boldsymbol{x_1}$ $(\boldsymbol{x} < \boldsymbol{x}_1)$ we have a *zero probability* (impossible event or result) while after $\boldsymbol{x_4}$ $(\boldsymbol{x} > \boldsymbol{x}_4)$ all possible results are achieved (certain event or result).

Binomial Law

It is based on the Bernoulli random variable (***p*** for success or achievement and ***q*** for failure or non-achievement). Its objective is to determine the probability of a precise result for an experiment of ***n*** *sub-experiments*.

Example 01

A *fake coin* is thrown with a probability of ***tail*** (***T***) realization equal to **0.4** (here ***tail*** corresponds to ***p*** or desired result). Therefore, the probability of making ***head*** (***H***) is equal to **0.6** (***q*** for head). We throw the coin five times in a row.

- What is the probability of having two heads followed by three tails (two-times head followed by three-times tails exactly)?

- What is the probability of having two heads and three tails?

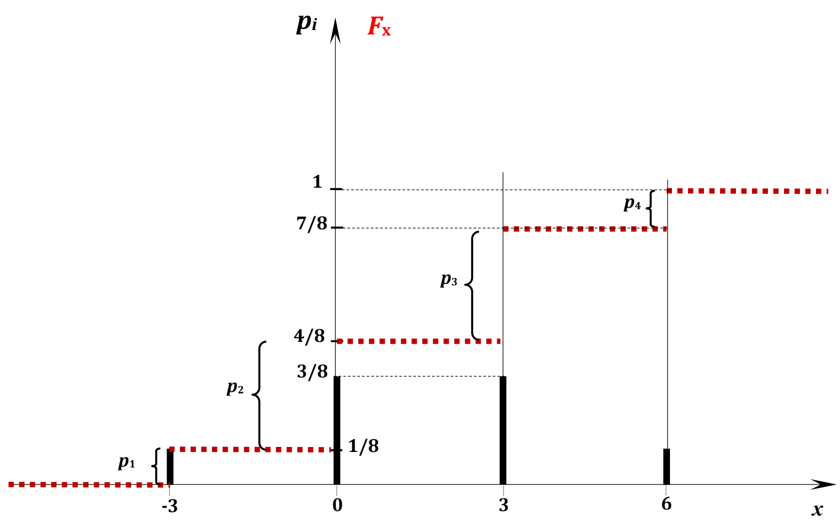

Fig. (6.1). Graphical representation of probabilities and distribution function of ***x***.

Solution

- For *the first question* and because the result of a throw is independent of the one before and after, we can easily calculate the probability by multiplying the probabilities for each throw:

$$p(HHTTT) = p(H)\times p(H)\times p(T)\times p(T)\times p(T) = 0.6\times0.6\times0.4\times0.4\times0.4 = 0.6^2\times0.4^3$$

If we are most interested in the ***tail*** event, then we have three tails. We can say we have ***k*** tails among ***n*** launches. Furthermore; if the event is not *tail*, it is necessarily *head*. So, for ***n*** launches, if we have ***k*** tails, we necessarily have (***n-k***) heads.

PART 2: STATISTICS

CHAPTER 7

Definitions and Calculations in Statistics

Abstract: The Statistical part was included in only one chapter given the ample details provided for the probability part. For this, the necessary terminology in statistics is presented well as it should be. Thereafter, the exploitation of the central tendency to determine the confidence interval and the risk of error according to different situations are presented in detail and with illustrative examples.

Keywords: Confidence interval, Central tendency and indicators, Relative standard deviation, Risk of error, Statistical series, Vocabularies.

INTRODUCTION

Statistics is the study of a phenomenon by collecting data, processing it, analyzing it, interpreting the results and presenting them, in order to make the data understandable by everyone [8-10]. It is both, a science and a method or a set of techniques.

Sometimes:

- The **S**tatistics: With a *capital* **S** to describe the science;
- The **s**tatistic: With *lowercase* **s** for a statistic.

It is applied (for applied statistics) to many areas, such as:

- **Geophysics:** Weather forecasts, climatology, pollution,… *etc*;
- **Demography:** Census of a population, which gives a photograph of it, for the purpose of representative surveys sampled when necessary;
- **Physics:** By making statistics on samples that provide a clear idea of the overall behavior of physical phenomena or systems;
- …... others.

We speak of *descriptive statistics* (our case), when we are interested in collecting and processing data for understanding; While we talk about *mathematical statistics* when we are looking to find estimators for future prevention.

A statistical series can be with a ***single variable*** or with ***several variables***.

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Notice: We can see the part of *statistics* as that of *probability*, because we have the same logic and the same laws. In probability, we work on non-existent phenomena, of which we want to determine the outcome according to the maximum probability of realization or non-outcome in the opposite case. In other words, we want to control chance. In Statistics, we work on facts, whose values we know and we try to take where there is a lack of information by probabilistic tendency (we have the same distribution laws in both parts). In probability, the achievement of an outcome is a percentage between the partial achievement and the total one (Number of favorable cases/Number of possible cases). In statistics, it is the -classical- percentage between elements (Number of elements in a category /Total number of elements).

SOME VOCABULARIES

Population

The set of individuals on which our statistics are based;

Individual

An element of the population;

Sample

Part of the population. It must be representative and sufficiently numerous. It is used when the number of the population is very large;

Example 01: If, for example, a statistical study is carried out on an automobile manufacturing company:

- *The population*: Automobiles;
- *The sample*: 100 automobiles for example, of which all models are included;
- *The individual*: One car.

Modality

This is the characteristic studied for the individual. It can be quantitative (defined as measurable) or qualitative (not measurable).

We can summarize that each statistical variable (also called random variable) follows the diagram below (Fig. **7.1**):

A STATISTICAL SERIES (WITH ONE VARIABLE) $\{(x_i, n_i)\}$

- x_i : The value of the statistical variable [8;11];
- n_i :The corresponding workforce (number of individuals for the same x_i)

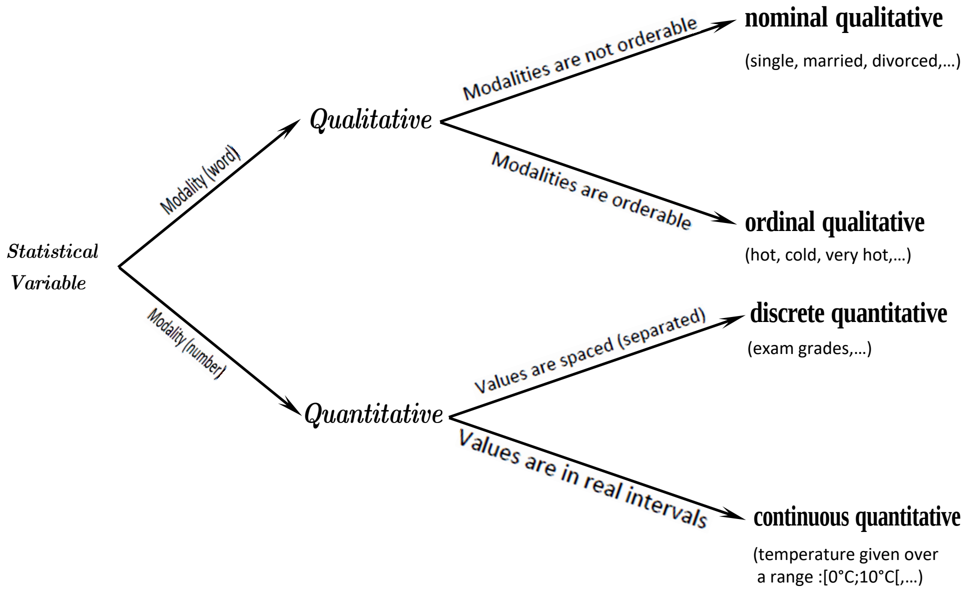


Fig. (7.1). Modality of the statistical random variable.

Example 02: We did a statistical analysis of **30** *students* about how *many siblings* (brothers and sisters) each of them has. We obtained (Table **7.1**):

Table 7.1. Statistical series of the 30 students.

x_i (N^{br} of siblings)	0	1	2	3	4	5	
n_i (N^{br} of students)	4	5	8	4	7	2	$\sum_{i=1}^{6} n_i = 30$

So we have 04 persons (students) who have no brothers or sisters at home. 02 only have five siblings.

Series of Probability Exercises

Series N° 01

Sets

Exercise 01

Determine ***x*** and **y** so that **I = J**, in the following cases [1-3]:

- **I** = {5; 2; 8; 6; 9} and **J** = {5; 8; *x*}
- **I** = {k; b; *x*; a; c} and **J** = {b; d; *y*; h; a}
- **I** = {19; *x*; *y*} and **J** = {24; 20; 7; 25; 19}

Note: *x* and *y* can be more than one element (subset).

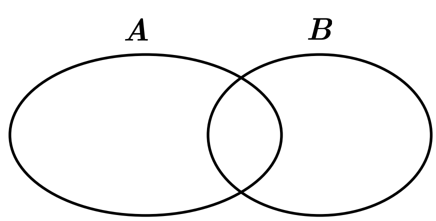
Exercise 02

Draw a VENN diagram that represents the set: **A**={2; 4; 7; 9; 13} and place the numbers **0** and **20** on this diagram.

- Complete the VENN diagram if we have both sets:

B={1 ; 3 ; 4 ; 6 ; 7 ; 11} **C**={4 ; 6 ; 9 ; 10}

Exercise 03



On the following diagram, hatch the empty parts:

- If $\mathbf{A} \subset \mathbf{B}$
- If $\mathbf{B} \subset \mathbf{A}$
- If $\mathbf{A} = \mathbf{B}$

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Exercise 04

And let three subsets of **S** be defined as follows:

Let the set **S**={0; 1; 2; 3; 4; 5; 6; 7; 8; 9}

A={1 ; 2 ; 3 ; 4 ; 5 ; 7} **B=**{*x/x* a number>4} **C=**{*x/x* an even number}

- Determine the following sets: $B \cap C$; $A \cup \ B \cap C$; $A \cup B \ \cap \ A \cup C$; A^c ; C^c ; $A^c \cap C^c$; $A \cup C \ ^c$; $A^c \cup C^c$; $A \cap C \ ^c$
- What do you notice?

Exercise 05

Out of **50 people**, **24** practice *Tennis* and **15** *Swimming*; **06 people** practice *both sports*.

- How many people do not practice any sport?
- We question a person at random. What are the probabilities of having
- Choose a person practicing *only one sport*.
- Choose a person who plays *Tennis*.

Exercise 06

A statistic made on **100 people** gave: **75** *people* have already owned a *European brand car*; **60** *people* have already owned an *Asian brand car* and **45** have already owned a *European* and *Asian car*.

- How many of the people questioned who *did not own* a car (neither European nor Asian)?
- How many of the people questioned those who have already owned a European car but not an Asian?
- Use the relations on the sets to solve this problem, and do a check with the VENN diagram.

Exercise 07

We give the results of the baccalaureate for the following different categories:

Result / Class	Obtained	Failed	Total	%
A		**938**		**28%**
B		**1036**	**2400**	
C				**15%**
D	**1978**			
Total			**9600**	
%	**65%**			

- *Complete the table*;

A student is asked at random:

- What is the probability that he is a **category B** student who ***obtained*** his baccalaureate?
- What is the probability that he is a **category B** ***baccalaureate graduate***?

Exercise 08

A survey of **500 students** gave the following statistics:

186 *students* study *economics*; **295** *students* study *statistics*; **329** *students* study *mathematics*;

83 *students* study *economics* and *mathematics*; **217** *students* study *mathematics* and *statistics*;

63 *students* study *economics* and *statistics*;

- How many students study the 03 modules together?

Exercise 09

A survey of **60 people** revealed that:

25 *people* read the newspaper **M**; **26** read newspaper **C**; **26** read newspaper **N**;

Series of Statistics Exercises

Series N° 7

STATISTICS

Exercise 01

Determine for the series below, the *mean*, the *mode* and the *median* [8, 9, 12, 14].

Series 1: 1 3 6 8 9 11 14 17 20 24 29

Series 2:

x_i	02	04	07	14.5	10.0	8.8	7.0	9.0	3.0
n_i	02	06	01	01	07	05	02	04	03

Exercise 02

Statistics made on **40** *employers* of a company about their *late arrivals* (in minutes) *of morning* entry are given in the table below:

Class	[A-B]	[B-C]	[C-D]	[D-E]
Center of the class	7.5	12.5	17.5	22.5
Workforce (Frequency)	15	10	06	09

1. Determine the *population*, the *random variable* and give *its type*;
2. If you are informed that the *lengths of the classes are equal*. Calculate the *length* of the class and define *the limits* of each class;
3. Calculate the *mean*, *standard deviation* and *coefficient of variation* and give your opinion on the dispersion of the series ;
4. Carefully draw the *histogram of the series* and mention the *corresponding polygon* on it.

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Exercise 03

A quality control inspector extracted from his database, *a sample* of **40** *weeks* where he noted *X*, the *number of work accidents recorded per week*. He obtained the following results:

2 0 4 2 2 1 3 2 0 5 4 3 2 4 5 6 6 4 2 0

3 4 4 2 6 2 4 3 0 4 3 4 3 3 5 5 4 2 2 1

- Complete the following table and draw the representative diagram.

Nbr of accidents per week	Frequency	Relative frequency	Cumulative relative frequency
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
Total			

Exercise 04

Let *X* be the *daily receipts* (*in dollars*) of a *small store*. A sample of size ***n***=**40** *days* was randomly selected which yielded the following results:

16.00 58.50 68.20 78.00 79.45 142.20 145.3 186.70 209.05 216.75

219.70 247.75 249.10 256.00 257.15 262.35 268.60 269.60 270.15 284.45

319.00 332.00 343.29 350.75 354.90 372.60 383.20 389.20 404.55 420.20

428.50 432.40 444.60 446.80 456.10 458.10 493.95 511.95 521.05 621.35

The number of classes to be formed is: $\boldsymbol{K} = 1 + \frac{10}{3}\mathbf{log}(\boldsymbol{n})$

The amplitude of each class equal to: $A = \frac{X_{\max} - X_{\min}}{K}$.

Note: The results for ***K*** and ***A*** must be rounded off with excess.

X (Recipe/Day)	Frequency	Relative frequency	Cumulative relative frequency
[… , …]	-	-	-
. . . .	. . . .	. . . .	. . . .
Total			

- Complete the following table and draw the representative diagram.

Exercise 05

Here are the distribution frequencies of a population.

Calculate the following:

- The *arithmetic mean*;
- The *mode* with explanation of the result;
- The *median* with an explanation of the result;
- The *first quartile* with an explanation of the result.

x_i	[1-1.5]	[1.5-2]	[2-2.5]	[2.5-3]	[3-3.5]	[3.5-4]
n_i	13	40	96	65	30	50

Series Correction Probabilities

SERIES 1

Sets

Exercice 01

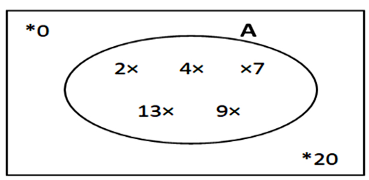
- ***I**J***={2 ; 5 ; 8} $\Rightarrow$ ***x***={2 ; 5 ; 8}
- ***I***={k ; b ; ***x*** ; a ; c} and ***J***={b ; d ; ***y*** ; h ; a} $\Rightarrow$ {k ; ***x*** ; c} = {d; ***y*** ;c} $\Rightarrow$ ***y***={k ; c} and ***x***={d; c}

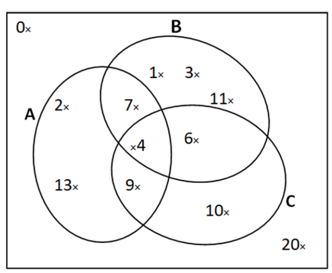
This solution is *not unique*, we can add on both sides, one element or more of our choice, but the same element (s).

- ***I***={19 ; ***x*** ; ***y*** } and ***J***={24 ; 20 ; 7 ; 25 ; 19} $\Rightarrow$ {***x*** ; ***y***}={24 ; 20 ; 7 ; 25}

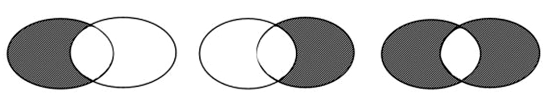
In this case also we have infinity of possibilities.

Exercice 02





Exercice 03



Exercice 04

- We define *B* and *C* in extension (i. e. enumeration)

B={5;6;7;8;9} ; *C*={0;2;4;6;8}

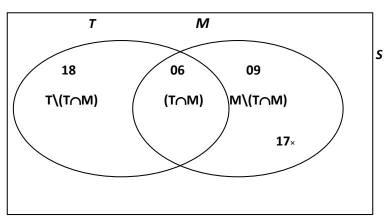
So : B∩C={6;8}

- A∪(B∩C)={1;2;3;4;5;7}∪{6;8}={1;2;3;4;5;6;7;8}
- (A∪B)∩(A∪C)={1;2;3;4;5;6;7;8 ;9}∩{0;1;2;3;4;5;6;7;8}={1;2;3;4;5;6;7;8}
  So : A∪(B∩C)= (A∪B)∩(A∪C)
- A^C=S\A={0;6;8;9}
- C^C=S\C={1;3;5;7;9}
- $A^C \cap C^C$={9}
- $(A\cup C)^C$=S\(A∪C)=S\{0;1;2;3;4;5;6;7;8}={9}
  So : $(A\cup C)^C = A^C \cap C^C$ (**1st** of De-Morgan)
- $A^C \cup C^C$={0;6;8;9}∪{1;3;5;7;9}={0;1;3;5;6;7;8;9}
- $(A\cap C)^C$=S\(A∩C)=S\{2;4}={0;1;3;5;6;7;8;9}

So : $(A\cap C)^C = A^C \cup C^C$ (**2nd** of De-Morgan)

Exercise 05

We draw the VENN diagram for the two sets. ***T*** for those who practice *Tennis* and ***M*** for those who practice *Swimming*.



- We have the total set of people questioned Card(*S*)=50

So, those who do not play any sport are: *S*\(*T*∪*M*) and their number=50-(18+6+9)=17.

- The *probability* of having a person playing ***only one*** sport is equal:

$$p = \frac{Card\left(T \setminus (T \cap M)\right) + Card\left(M \setminus (T \cap M)\right)}{Card(S)} = \frac{18+9}{50} = 0.54$$

- The *probability* of having a person playing ***Tennis*** is equal:

$$p = \frac{Card\left(T \setminus (T \cap M)\right) + Card\left(T \cap M\right)}{Card(S)} = \frac{18+6}{50} = 0.48$$

Exercise 06

As done for the previous exercise, we name those who owned a *European car* by ***E*** and those, an *Asian car* by ***A***.

So, we have: Card(E)=75 ; Card(A)=60 ; Card($J \cap A$)=45

- The number of people who did not own a car (neither *European* nor *Asian*) is in total *minus* the number of those who have ever owned a car or two.

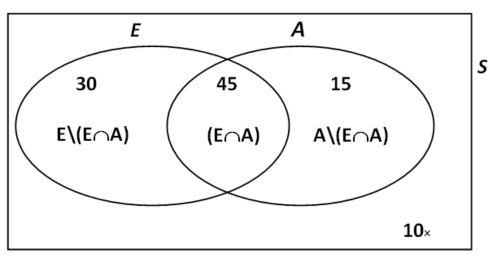
$$Card(NC) = Card(S) - Card(E \cup A) = Card(S) - \left[Card(E) + Card(A) - Card(E \cap A)\right]$$

$$= 100 - (75 + 60 - 45) = 10$$

- The number of people who have owned a *European* car but not an *Asian* car is defined by: Card($E \setminus (E \cap A)$)

So: $Card(E \setminus (E \cap A)) = Card(E) - Card(E \cap A) = 75 - 45 = 30$

- Verification with the VENN diagram:



Exercise 07

The table can be completed in several ways. After all the calculations, we arrive at the following values in the table:

Series Correction Statistics

Series 7

STATISTICS

Exercise 01

We are asked to calculate for the series below, the *mean*, the *mode*, the *median* and the *extent*.

Series 1: 1 3 6 8 9 11 14 17 20 24 29

$$\circ\ \bar{X} = \frac{1}{N}\sum_{i=1}^{11} x_i = \frac{1+3+6+8+9+11+14+17+20+24+29}{11} \simeq 12.91\ ;$$

- ○ $Mod = 0$ (No mode : all $\boldsymbol{x_i}$ have the same frequency 1);
- ○ $Med = 11$ (The value 11 divides the series exactly into two equal parts. We have an odd number of values);
- ○ $Ext = x_{i\max} - x_{i\min} = 29 - 1 = 28$

Series 2

x_i	02	04	07	14.5	10.0	8.8	07	09	03
n_i	02	06	01	01	07	05	02	04	03

$$\circ\ \bar{X} = \frac{1}{\sum_{i=1}^{9} n_i}\sum_{i=1}^{9} n_i \times x_i = \frac{2\times2+6\times4+1\times7+1\times14.5+7\times10+5\times8.8+2\times7+4\times9+3\times3}{2+6+1+1+7+5+2+4+3}$$

$$= \frac{222.5}{31} \simeq 7.18$$

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The series should be ordered before calculating the remaining parameters. We get the new representation:

x_i	02	03	04	07	8.8	9	10	14.5
n_i	02	03	06	03	05	04	07	01

- $Mod = 07$ (**x_i** =10 has the greater frequency);
- $Med = \frac{7+8.8}{2} = 7.9$ (We have an even number of **x_i**. we take the mean value of the two middle ones);
- $Ext = x_{i\max} - x_{i\min} = 14.5 - 2 = 12.5$

Exercise 02

We have the following statistics:

Class	[A-B]	[B-C]	[C-D]	[D-E]
Center of the class	**7.5**	**12.5**	**17.5**	**22.5**
Workforce (Frequency)	**15**	**10**	**06**	**09**

1.

- *Population*: The workers
- *Statistical variable*: The time
- *Its Type*: Quantitative (counted in minutes and therefore quantified by numbers)

2.

The lengths of the classes are the same ⇒ **(*B*-*A*) = (*C*-*B*)** $\Rightarrow C = 2\times B - A$

And we already have: **(*B*+*A*)/2=7.5** and **(*C*+*B*)/2=12.5**
$\Rightarrow\ B+A=15\ \ and\ \ C+B=25$

$$\Rightarrow\ B=15-A\quad et\quad 2\times B-A+B=3\times B-A=25$$

$$\Rightarrow\ 3\times(15-A)-A=25\quad\Rightarrow\quad 45-4\times A=25\quad\Rightarrow\ \boxed{A=5}$$

$$B=15-A\ \Rightarrow\ \boxed{B=10}$$

$$similarly,\ we\ find:\ \boxed{C=15}\ ;\ \boxed{D=20}\ ;\ \boxed{E=25}$$

So, each class length equals: **05*mn***

3.

$$\overline{X}=\frac{1}{\sum_{i=1}^{5}n_i}\sum_{i=1}^{5}n_i.x_i=\frac{1}{40}\left(15\times 7.5+10\times 12.5+6\times 17.5+9\times 22.5\right)=13.625\,mn$$

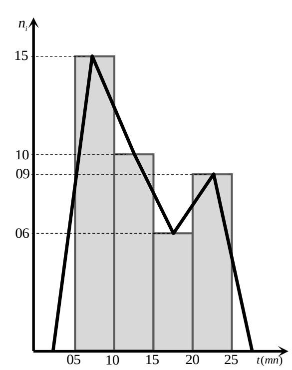
$$\sigma=\frac{1}{\sum_{i=1}^{5}n_i}\sum_{i=1}^{5}n_i.x_i^2-\overline{X}^2=\frac{1}{40}\left(15\times(7.5)^2+10\times(12.5)^2+6\times(17.5)^2+9\times(22.5)^2\right)-(13.625)^2$$

$$\simeq 5.86\,mn$$

$$CV=\frac{\bar{X}}{\sigma}\simeq 0.43<0.5$$

The value of ***CV*** is less than **0.5**, it means that the series is weakly dispersed. But, its value is ***close to 0.5*** is it anyway dispersed a considerable degree.

4.



CONCLUSION

At the end of this first version of the book, I hope that the reader (student, teacher or other) has benefited from the content without difficulty. I repeat here that I presented the part on probabilities first because of its difficulty compared to that of statistics among students according to my experience teaching the module. In addition, probability calculations are essential for technical specialties more than economic ones where the part of statistics is the most encountered. I tried to detail in a chained way the theory, the process of the calculation and the exploitation of the definitions by discussed examples in order to show why we study this chapter or the other. I showed that the laws of probability are those of statistics by difference of domains only, where for the probabilities we make anticipations even of the problems to be studied themselves. *i.e*, we study the problem mentally and we imagine all its outcomes based on their probabilities, then we fix our decisions depending on the weighting or the rarity of the outcome sought (favorable case). On the other hand, in statistics, the problem and its outcomes already existed and the weightings are determined from the samples which change form and type according to the problem and the objectives. Therefore, mastering one part serves with excellence to master the other, despite the recommendation to start with the part of probabilities boosted by the factor of imagination. I hope that in future versions, enrichment in quality and content will soon be assured.

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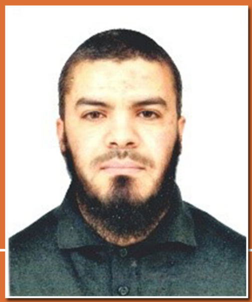

Horimek Abderrahmane

Prof. Horimek Abderrahmane is a full professor at the University of Ziane Achour Djelfa -Algeria- since December 2012. In 2001, he obtained his first diploma as a state engineer in energy mechanics from the National Institute of Mechanical Engineering (Ex INGM) of Boumerdes. In 2005, he obtained a specialized engineer diploma in thermal engines from the Algerian Petroleum Institute of Boumerdes. In 2006, he obtained his magister diploma, and in November 2014 his doctorate of science degree, both in energetic mechanics specialty from Blida University 1 in Algeria. Currently, Dr. Horimek is a lecturer. He received several practical internships. He has more than 60 scientific collaborations (publications, keynotes, international and national conferences and book' chapters). He was a member of several scientific committees of international seminars. His research areas are fluid mechanics, heat transfer, non-Newtonian fluids in laminar, transitional and turbulent regimes. The current book is about the "Probability and Statistics" module intended for undergraduate students. By exploiting his knowledge as a doctor of mechanical engineering, he tried to present the content in a more practical than theoretical way, without stripping away the essential definitions but with necessary and sufficient details.